Anatomical and Mathematical Researches into the Laws of the Variations of Brain Volume and their Relation to Intelligence

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[Editor's note: This remarkable treatise, unparalleled in the history of Science, was crowned by both the French Academy of Sciences and the Anthropology Society of Paris.]

Anatomists are today in agreement in recognizing that it is principally in the inequalities of the development of the nervous system where one must seek the origin of the differences which differentiate living beings.

It was hardly a century ago that many philosophers considered all men as being endowed with a uniform nature and equal intelligence, and attributed their differences to the unequal education which they received. However, any attentive study of the diverse races that inhabit the globe will demonstrate the inanity of this belief. Such a study will show that the physical and intellectual differences which distinguish the various representatives of our species are profound and are produced in the newborn. Each man, it can be clearly perceived, already possesses behind him a long past. The ensemble of physical, intellectual, and moral characteristics which manifest themselves at some point exists already in the germ and represents the heritage of the past. A person's aptitudes are the result of all the successive modifications undergone by his long series of ancestors. The ages weigh on him like a load, and there is no way to remove it. With his beauty and his ugliness, with his virtues and his vices, man conveys the traces of a past which is not his own. It requires the work of centuries and not that of one day to bring about by education the level of the European nations to savages similar to our ancestors of geologic ages, people who were unaware of metal, ignorant of agriculture, killed their aged parents and possessed no law but that of the strongest.

It is very easy to foresee that the quite profound intellectual differences that exist between the diverse human races and between
individuals of the same race will correspond to the not less profound anatomical differences. Only today has science commenced to the study of these differences, but it is not yet prepared to determine all of them. Nevertheless, it already has succeeded in placing into evidence some of the most important ones and, more particularly, it has shown that among such are those found in the state of the development of the brain, that is to say, the organ of intelligence.

The hour has not come yet where the materials one collects are of sufficient number that, in bringing them together into a vast whole, one can disentangle from them with certainty the pertinent laws. We expect, however, that a thorough analysis will enable us to present those that are these laws.

The first time that we try to effect a work synthesizing the various anthropological materials that science possesses and those which we have attempted to assemble and combine, materials whose aggregate appears to the observer in a form uniquely filled with volumes of numbers, to us it seems at first impossible to apprehend any affinity among the values of such seemingly strong dissimilarities. However, the application of mathematical methods, new to craniology, has allowed us to extricate the concealed relations which, in fact, are really present. Materials more complete than those that we already have in our hands may perhaps modify the details of some of the results to which we've been conducted; however, they will not have to await methods that yield meaningful results.

The researches that will be set forth, from which this Paper is devoted, required more than a year of persevering work. We have urgent motives for their undertaking. Before writing certain chapters of the second volume of my book: l'Homme et les sociétés, leurs origins et leur histoire, we had need of precise records explaining the differences existing between men and the manner in which these differences can grow or give way. Anthropology alone was able to furnish us these records. It is from her that we have requested them.

## I. Method of Study

Comparison of the measurements obtained on a large number of skulls. The method of averages. - Artificial results that they furnish. - Impossibility of taking a useful average between dissimilar values. - The method of groupings by series. - Its simplification by graphic processes. - Examination of the graphic methods normally used. - Binomial curves and applications of the binomial formula. - New curves permit groupings by series to be expressed. - Different examples of the applications. - Possibility of predicting phenomena revealed by these curves whenever one knows their equation. - Application to the parabolic curve which expresses the distribution of the population in France.

Measurements obtained on skulls of the different human races are so variable within each race from one individual to another that, in order to draw any conclusions whatsoever from the comparison of such isolated cases, scientists have been led to group together a certain number of observations and then divide their sum by the number of observations. It is thus by obtaining averages, which represent a sort of intermediate value for all these observations, allowing one to organize them, that one can then make comparisons between them. The greater the number of subjects within each group, the more the averages obtained can be considered as expressing completely the state of these different groups.

This method of expressing and comparing results, called the method of averages, is the only one that has been used so far in craniometry. It is, moreover, the method most widespread in statistics.

Useful when it contents itself to determine the average of a group of values which differ little, such as, for example, the observations of an astronomical phenomenon, this method will prove to be entirely illusory when it concerns itself with comparing values which differ greatly. The values produced for representing the average length of life, average height, etc. of a great number of subjects possessing very different ages and heights are artificial values which seemingly ought to represent the numbers which one observes the most frequently, but which represent, on the contrary, those which are observed the most rarely. When one says, for example, that the average length of life in a country is, let's suppose, forty years, it immediately
appears that this is the period in life where the great majority of individuals die: but observation demonstrates, quite contrarily, that it is the minority who die at this age. It is in extreme infancy and in old age where the greatest mortality is encountered, and by no means can these periods be indicated by the average.

Most averages provide results just as misleading. The value given for individual consumption of meat in France, for example, is a value obtained by combining, like in the preceding cases, results from entirely different subjects: wealthy class individuals who eat a lot of meat, the less comfortable classes who do not eat much meat, and inhabitants of certain rural areas who do not eat any meat at all. The resulting average is naturally an absurd value.

These general averages which make no distinction between entirely dissimilar elements-subjects large and small, rich and poor, age and sex of individuals, conditions and different manners of living-are useful for indicating en bloc the variations that show the state of civilization reached by a considerable mass of individuals, the influence of the milieu, etc.; but, these averages are ineffectual in furnishing us anything but the most flimsy pieces of information about the diverse variations which occur within the groups that comprise this mass. Now, it is precisely these pieces of information which are often the most important to obtain. The averages of statisticians are generally just as useless to an economist or a philosopher as the knowledge a hatmaker is able to attain of the average size of all the hats sold by him; by taking this measure as a guide when renewing his assortment, the hats made according to this information will not prove to be of service to but a quite restricted number of individuals.

The criticism that I have seen necessary to formulate against the process of general averages is applicable, in my opinion, to the majority of averages employed in anthropology. I am able to well concede its extreme precision in describing the average man in a group composed of subjects of like descent, living in similar surroundings, and therefore more or less homogeneous. However, I do not know how it can allow for the average man of a race, and still less, of course, the average man of all the races, like the hybrid
being created by Quételet. With his average qualities, he represents a sort of effaced type, of which one will perhaps not find any living representative on the surface of the globe. There is, in fact, no average that can be established between beauty and ugliness, between smallness and grandeur, between knowledge and ignorance, in short, between any quite dissimilar elements.

That which I have seen to express about the average man of a race I shall now apply equally to the average skull of a race. The present method employed in craniology, which consists of adding the volumes of skulls taken at random of a certain number of individuals, then dividing the total thus obtained by the number of skulls measured, may certainly yield in certain cases generally useful results, but these results are always extremely insufficient. Just like in the preceding case, in effect one compares quite dissimilar values, among which there isn't any relation that exists. There is not any average one can accept between skulls measuring 1200 cubic centimeters in volume and skulls which measure 1900 cubic centimeters, nor is there any acceptable average between the intelligence of a Cuvier and that of his porter. The intermediate value obtained by subtracting from the largest skull in order to add to the smallest skull is entirely fictitious; the largest skulls and the smallest skulls are each contained in separate categories, and it's not possible to compare any but those belonging to the same category. Moreover, one might arrive at the fact, as I shall further point out, that the average skull volume of two different races is identical, although one race may present more voluminous skulls than the other, and for this reason is superior to it.

Returning to some of our previous examples, let us see how by adopting this fundamental principle of only making comparisons between values belonging to the same class it is possible to obtain information which is greatly superior to that provided by averages.

Suppose, for example, that what is in question is determining the individual consumption of meat in France, that is to say, of knowing how the French population in reality feeds itself; we enumerate in different columns those individuals who do not ever eat meat, those who eat it occasionally, those who make it their
nourishment habitually, etc. By comparing then these elements of a like nature, we arrive at results entirely different from those fictitious averages that statistics gives us.

When it is a matter of comparing skulls, the procedure we'll employ will be quite the same as above. Instead of lumping together the largest skulls and the smallest skulls which each race contains, then adding them up and deriving the average, we shall classify them by groups of fixed capacities and determine how many skulls of each race belong to each group. By placing together, for example, skulls measuring 1300 to 1400 cubic centimeters, those of 1400 to 1500 cubic centimeters, etc., and then obtaining the total for each of these groups, we are able to understand how many large, medium, and small skulls a race contains. Given that among the examined subjects there is only a limited number of extremely large skulls or very small skulls, this number stands out immediately in evidence, whereas with the method of averages it becomes entirely erased.

The only inconvenience of the preceding grouping approach is that it will present in the final analysis several numerical amounts instead of just one, as occurs with averages; or, when it is a matter of comparing a number of different elements, for example, the skulls of the various races, the simultaneous study of several amounts might be considerably difficult.

But there exists a method (unused till now in craniology)-the graphic method, which allows for the substitution of numerical totals with lines, and it conveys in the clearest manner those relations which in a substantially less clear fashion numerical totals express as occurring.

In order to graphically represent the variability of a changeable value, such as height and weight, for example, authors have generally made use of graphic curves designed by Quételet and other statisticians which are named binomial curves, because for a great number of phenomena their mathematical model is represented by Newton's law of binomial coefficients. On the horizontal abscissa one places the dimensions which are relevant to the series: weights, heights, age at death, etc., of which it's a question of expressing the
variations, and on the vertical ordinates the amounts indicating the series' variations. The curve thus obtains the aspect of an inverted $\mathbf{V}$ when drawn out from the graph's lower lateral edges.

The mathematician Quételet has established this most curious fact: that the variations of certain dimensions, like weight and height, for example, are able to be expressed by the binomial formula of Newton. In taking 10,000 30-year-old men, one is able, by knowing the extreme weights found in this group, to determine the men's respective weights with the formula which regulates the intermediate class. Likewise the men's heights will be uniformly ranged along the division of ordinates by this same formula.

The same formula also enables one to express the size and growth of men at different ages, and is applicable to many more physical phenomena, such as the extent of mistakes committed in observing a star passing across the meridian, errors in aim while shooting at a target, etc. The curve which represents the errors that one commits while observing the passing of a star or while shooting a great number of times at a target, for example, is extremely regular and very symmetrical for each part of the ordinate, which indicates by its height the largest number of errors committed.

The application of the preceding formula is far from being as frequently useful as Quételet desired. In reality, a case is hardly ever witnessed where the increase or decrease of a dimension is made in a symmetrical fashion about a certain amount. With most cases one obtains very irregular curves.

Besides, even though some curves are quite regular, those thus obtained are difficult to compare with one another, and consequently do not permit one to immediately perceive important relations that may present phenomena that they express. In conveying, for example, with this curve form the variations of skull volume of the various human races, one obtains shapes calling to mind the aspect of a series of complex inverted V's, whose interpretation is much more complicated than that of the numerical values that they have replaced.


Belgian statistician Adolphe Quételet (1796-1874)

After having renounced the employment of binomial curves and after also trying out diverse graphic processes, such as polar diagrams, cartograms, etc., I came to recognize that none of these methods satisfy the double goal that I myself propose: first, an ability to obtain curves whose main aspect clearly indicates the composition of the groups which are to be expressed, and secondly, of being able with their comparison to immediately perceive the connections and resemblances which they present. I have therefore been led to invent a system of special curves in which the abscissas are equidistant, while the ordinates express by their height the values which depict the variations, and by their spacing the centesimal affinity of these variations.

Just casting a glance at these curves is sufficient for one to recognize that they are exceedingly clear and are of an extremely easy construction. So that you will understand this, I shall provide an example. Suppose one wants to convey in the language of graphs the variations one observes in the circumferences of the heads of modern, middle-class Parisians. Of the observations that I will discuss in another chapter, and which are supported by the measurement of a considerable number of heads, I have proven that the verified variations, adjusted to the total of 100, are able to be expressed by the following table:

## Circumference in centimeters

## Percentage <br> of each circumference per 100 heads

The axis of the abscissas (Plate VII, Curve No. 2) is first divided into 100 parts, divisions which exist ready-made by the millimeter on quadrille graph paper; the axis of the ordinates is likewise divided into sufficiently numerous equidistant parts for one to comprehend all the observed variations pertaining to the preceding case of 52 to 62.5 centimeters.

The ordinates that we set up on the axis of the abscissas must, as I have said in defining these curves, express by their height the values of which it's a matter of depicting the variations and, by their deviation, the percentage of these variations. In other words, the ordinates must correspond by their height to the number appearing in the preceding column of the table and, by their separation, correspond also to the number appearing in the next column. In the above example relating to the head circumference of modern, middle-class Parisians, the first ordinate finds itself 0.6 mm from zero; the second ordinate exists 1.9 mm from the preceding ordinate and, consequently, $0.6+1.9=2.5 \mathrm{~mm}$ from 0 ; the third ordinate is met with at 6.2 mm from the preceding, that is to say, $0.6+1.9+$ $6.2=8.7 \mathrm{~mm}$ from O , etc. The position of each ordinate depends therefore, as can be seen, on that of the preceding ordinate, and all the ordinates consequently enjoy a close dependence amongst
themselves. The height of these ordinates is naturally limited and determined by the range of numbers drawn upon the $y$-axis. The three preceding totals-0.6, 2.5 and 8.7-dwell therefore on horizontal lines which emanate from the numbers 53,54 and 55.

In making use, as I have indicated above, of quadrille graph paper marked off in millimeters, the ordinates and abscissas exist entirely delineated beforehand. The operation of creating the graph amounts to marking by pen on the axis of the abscissas the number of millimeters that exist between each ordinate and then to register, on the ordinate that lies above this mark, its height. The marked down points are then connected by a continuous line, and the operation is concluded.

The preceding description is actually substantially longer than the operation itself. Performing it but a single time is all that is necessary for one to immediately recognize its extreme simplicity.

When it is a question of determining skull volumes, heights, mortality, etc., the manner of these graphs' construction is identical to that described above. Along the axis of ordinates one writes down numbers corresponding to the volumes, heights, or ages that call for expression; meanwhile, the axis of the abscissas is always divided into 100 parts, and the ordinates possess a deviation proportional to the importance of the group that they represent.

Although their deviation is quite variable, the ordinates appear at first glance equidistant on the figures because one has availed himself, in order to simplify their construction, of quadrille paper, but it only takes a moment to understand that in reality their deviation is highly variable. It is also only this deviation which makes for understanding the centesimal proportion of the phenomenon. It is sufficient, for example, to count how many millimeters exist horizontally between the points where the curve cuts through the abscissas corresponding to skulls possessing 56 and 57 centimeters in circumference in order to know out of 100 skulls how many there are that have this circumference. It is evident that the larger the number of millimeters is, the more the curve tends to become horizontal; by contrast, the smaller the number, the more the curve
tends to be vertical. The curve contained between two points forms, in effect, the hypotenuse of a triangle whose vertical side has, as one can easily see, a constant height, and a horizontal side whose length continually increases the more the ordinates deviate. In examining curves with points that express very small groups-for example, the percentage of individuals possessing very small skulls-one sees the curves at these points becoming nearly vertical.

The property that these new curves possess, of immediately construing by their aspect the variation in hundredths of the elements existing in the midst of a given group, has led me to designate these curves as centesimal curves or curves of series. Instead of making no distinction between, as averages do, very different results, they provide on the contrary a very faithful picture of the smallest differences that the elements of a group present, from which one can understand the factors. Even if there is only one individual out of 100 possessing a certain cranial capacity or height, the curve will indicate it immediately, whereas averages will not even permit one to suspect its presence.

The importance of these indications will be proven to us when, in studying the variations of skull volumes, we see that it is far more in the more or less greater percentage of voluminous skulls that races contain than in the average capacity of their skulls that resides the superiority of some races relative to others.

It may sometimes occur that the variations one sees expressed present themselves only one time out of $1,000,10,000$, or 100,000 observations. In this instance the fraction conveyed in hundredths is too small to be expressed on a graph constructed on a very small scale; such a case might present itself with respect to extreme regions of some curves. One may easily remediate this situation by constructing this part of the curve on another sheet of paper possessing a scale of 10,20 , or 100 times larger, totally like the way topographers, after having drawn the map of a country to a given scale, draw, in a corner of this map, different parts, cities, towns, etc. in which they wish to render more of the details. With the cases that we have had to examine in this study, these supplementary constructions have been unnecessary.

While making understandable the percentage which comprises diverse elements in the constitution of a given group and also permitting one to compare with the greatest facility different groups from one another, these curves immediately furnish as well the averages-the statistic which is habitually used. When the curve has been constructed with a sufficient number of elements, the average exists exactly at the conjunction point of the curve with the ordinate that divides the axis of the abscissas into two equal parts. For this reason I have named this ordinate the axis of the averages. The reader can easily convince himself, by examining the curves of the height of the French and Italian population, that the averages provided by calculation and by the curve are nearly identical.

Besides the advantages that I have already signified, our curves of series permit us to bring together without confusion a large quantity of information in a small space. One may satisfy himself of this fact by examining Plate I, where I have intentionally assembled different documents which call for a large number of separate total amounts in order to be numerically rendered. They are:

1) The distribution of the population in France, that is to say, out of one million inhabitants, how many individuals there are of each age; 2) the tendency to crime of each age; 3) the variations of the height of newborns; 4) the variations of the weight of newborns; 5) the height of conscripts in France; and 6) the height of conscripts in Italy.

To the various amounts joined together in this manner on this chart (Plate I graph) measuring 10 centimeters on each side, I have clearly been able, without harming its clarity, to also add much additional information.

In examining these different curves, which convey quite different statistical facts, one is struck by their extreme regularity. The curve of the grouping of the population, for example, noticeably depicts within the longest part of its distance (from 0 to 90 years) a parabola of 20 millimeters about the focus.


# Plate I. - Application of curves of series, expressing diverse anthropological phenomena. 

1. Distribution by age of the French population ${ }^{1}$.
2. Inclination to crime according to age ${ }^{2}$.
3. Height of newborns ${ }^{3}$.
4. Weight of newborns ${ }^{4}$.
5. Height of adults in France ${ }^{5}$.
6. Height of adults in Italy*.
${ }^{1}$ From the numerical totals published by the Annuaire du bureau des longitudes (1858). -- ${ }^{2}$ From the numbers provided by Quételet in his Anthropométrie, originating from the officially rendered records of the Ministry of Justice, from 1826 to 1844. -${ }^{3}$ and ${ }^{4}$ From the records published in 1878 in the Annales de demographie, by the director of the Children's Hospital of the Seine. -- ${ }^{5}$ From the data provided by Bertillon in the latest edition of Nysten's Dictionary. -- *From the official records of the Italian government (La Démographie italienne, Rome, 1878).


## Plate I description and explanation (continued)

The $1^{\text {st }}$ scale on the left side (outside) is the scale of French and Italian conscripts measuring from 135 to 190 centimeters in height. In this scale, 2 millimeters $=1$ centimeter.

The $2^{\text {nd }}$ scale on the left side (inside) is the scale of years for the distribution of the population and for the inclination to crime. It goes from 0 to 100 years. In this scale, 1 millimeter = 1 year .

The $1^{\text {st }}$ scale on the right side (inside) is the scale of the height of newborns, from 36 to 56 centimeters. In this scale, 1 millimeter $=1$ centimeter.

The $2^{\text {nd }}$ scale on the right side (outside) is the scale of the weight of newborns in kilograms, from 1.3 to 4.6 kilograms. In this scale, 5 millimeters $=100$ grams.

With the spacing of the ordinates being proportional in our curves to the variations of the observed phenomena, in order to know how many subjects there are, for example, of a given height out of 100 newborns, it suffices for one to count the number of millimeters contained horizontally between the points where the curve intercepts the horizontal lines corresponding to the numbers of the scale indicating the given height. Let us say I wish to learn how many subjects there are out of 100 measuring from 51 to 52 centimeters in height; all I need do is count how many millimeters there are horizontally between the points where the curve cuts through the horizontal lines corresponding to the numbers 51 and 52. The 6 millimeters existing between the two points represents the sought after number. In other words, out of 100 newborns there are therefore 6 whose height measures from 51 to 52 centimeters.

The upwardly pointing vertical line located right at the middle of Plate I and which is labeled axe des moyennes intercepts each curve at a point that possesses the property of exactly representing the average of the numbers in question, under the simple condition that the series upon which one operates is sufficiently numerous. For example, the number given for the average height in France and Italy, obtained from statistical documents, is 164 and 162 centimeters respectively. These values are precisely the ones indicated by our curves, with an error of about 2 millimeters, that is to say, entirely negligible. In realizing the construction of the curve, one easily comprehends that the point where it intercepts the axis of the averages can only be modified by the will of the operator.

All these highly regular curves truly must be able to be expressed by an equation, perhaps not easy to discover, but which, once determined, will permit us, with some intermediate factors, to calculate the others and to solve problems such as this one: Knowing that the skull capacities of a race range from 1300 to 1900 cubic centimeters and, on the other hand, knowing how many individuals there are of this race possessing skulls 1200, 1300 and 1400 cubic centimeters in size, calculate how many there are of all the skull capacities falling between 1400 and 1900 cubic centimeters.

In order to prove that the solution to such problems is hardly chimerical, I shall now show that, thanks to the understanding of the equation of the curve which represents the distribution of the population in France, it is possible to know how many individuals of any age there are in this country. A well-known property of the parabola, that the squares of its ordinates are between them like the corresponding abscissas, permits us to easily verify that the curve which represents the distribution of the population in France according to age is for all intents and purposes within the longest part of its distance (from 0 to 90 years) a parabola. The analytic equation of the parabola:

$$
\text { (1) } y^{2}=2 p x
$$

from which one can deduce

$$
\text { (2) } p=y^{2} / 2 x
$$

permits us to immediately determine the parameter $(p)$ and consequently the focus of this curve. It is sufficient to take for any abscissa the height of the corresponding ordinate proceeding along the horizontal line up to the number 90 (corresponding to 90 years) and to replace the letters ( $\boldsymbol{x}$ and $\boldsymbol{y}$ ) by their values in equation (2) in order to find that $p=40$.

With our curve being a parabola and its parameter known, we can now solve equation (1) with respect to $x$, determining mathematically without any graphical construction whatsoever the number of inhabitants in France of all ages between 0 and 90 years old. It turns out that a phenomenon in appearance as
complicated and randomly formed as the number of individuals of different ages that a country contains can be expressed by this very simple equation:

$$
\text { (3) } x=y^{2} /(2 \cdot 40)
$$

The following application of this formula by means of an example shows its great accuracy and usefulness. Let us say that we wish to determine how many individuals there are in France from 35 to 40 years of age. If we refer back to the definition of our curves, which sets forth that the abscissas are equidistant while the ordinates are separated by intervals that express in hundredths the percentage of the value relating to the understanding and distinguishing of the variations, we see right away that it is sufficient to know how many out of 100 individuals, hence out of one million and consequently for the entire population, there are from 35 to 40 years of age by seeking out the millimetric deviation of the corresponding ordinates to the stated ages. The simple disposition of the adopted scale clearly shows, which is moreover immediately verified on the figure and without understanding any of the elements of the curve, that the heights of the ordinates corresponding to 35 and 40 are 55 and 50 millimeters respectively. Solving equation (3) for these two values of $y$ and then subtracting one from another, we obtain the number 6.56 , which tells us that mathematically there are 6.56 individuals out of 100 who are 35 to 40 years old. For one million there would therefore be 65,600 people of this age; and, by knowing the number for one million, a simple operation of arithmetic will allow us to determine the number of such individuals for the entire population.

The number 6.56 found by the above calculation is almost exactly the same as that provided by the tables of statistics. Now, in order to demonstrate that this is not a mere coincidence, I provide below the comparative table of the distribution of the French population, the values in this table being derived from the best statistical works and from my formula. As one can see, the correspondence between the values is remarkable: it is only in the extreme region (80 to 100) that we observe a difference, though very small.

## Population of Each Age in France (per 100 individuals)



Age
From 0 to 5 years
10.99
9.80
9.37
9.02
8.54
7.88
7.22
6.62
6.03
5.45
4.84
4.19
3.51
2.75
1.91
1.14
0.52
0.17
0.05
100.00

|  |  | 10.99 | 10.94 |
| :---: | :---: | :---: | :---: |
| ---- 5 | 10 | 9.80 | 10.31 |
| ---- 10 | 15 | 9.37 | 9.69 |
| ---- 15 | 20 | 9.02 | 9.36 |
| ---- 20 | 25 | 8.54 | 8.44 |
| ---- 25 | 30 | 7.88 | 7.81 |
| ---- 30 | 35 | 7.22 | 7.19 |
| ---- 35 | 40 | 6.62 | 6.56 |
| ---- 40 | 45 | 6.03 | 5.94 |
| ---- 45 | 50 | 5.45 | 5.31 |
| ---- 50 | 55 | 4.84 | 4.69 |
| ---- 55 | 60 | 4.19 | 4.06 |
| ---- 60 | 65 | 3.51 | 3.44 |
| ---- 65 | 70 | 2.75 | 2.81 |
| ---- 70 | 75 | 1.91 | 2.19 |
| ---- 75 | 80 | 1.14 | 1.56 |
| ---- 80 | 85 | 0.52 |  |
| ---- 85 | 90 | 0.17 | 1.02 |
| ---- 90 | 100 | 0.05 |  |

Number of individuals of each age calculated from the formula of the curve:

$$
x=\left(y^{2} / 2\right) \cdot 40
$$

It is most curious to see phenomena so variable in appearance as the number of individuals of each age that a country contains be distributed in accordance with a law so mathematical. This outcome, though, will undoubtedly be the same with the majority of events if we are able to subject them to calculation. Now, nothing seems more fortuitous than the birth or death of an individual. They are, however, the results of invariable necessities, and, in order to change by one second the time of day at which a person dies, it is necessary to change the entire series of antecedent events which, without such action, have already summoned and impacted us. Though man is powerless to ascertain the causes of events, at least he is able to show that nothing is more immutable than the eternal forces that guide the course.

Curves destined to express the variations of skull volume, all being very regular, generally possess an angular aspect that curves of other phenomena do not have. This comes about because for height, mortality, etc. one usually operates on a considerable number of individuals, allowing the module of the grouping (a few centimeters for height, a small number of years for population) to be quite small, whereas for skulls, their number being relative limited, it is necessary that the module of the chosen grouping (100 cubic centimeters) be large enough in order for each group to find itself composed of a sufficient number of observations. If we possess a much larger number of skulls, we might be able to form groups only differing by 50 or even 25 cubic centimeters in capacity; the sides of the curve will therefore become more numerous, and the angular aspect mentioned above will disappear, the same as when the sides of a polygon inscribed in a circle are sufficiently multiplied, the finished polygon differs very little in appearance from the circle's circumference.

Every time that the series of skulls upon which one operates are completely homogeneous, that is to say composed of elements of the same origin, the curves that one obtains are very regular; given such regularity any deformation of the curve indicates itself immediately, such as when one commits an error in calculation or when the series which one is working on is composed of heterogeneous elements (for example, skulls of different races or sexes that have been accidentally mixed together). In fact, I highly recommend this very simple means-i.e., searching for any irregularities in the appearance of the curve-in order to discover whether, in a series of skulls that one does not have before his eyes, but which one does possess the capacities, many races and many sexes have been mixed together. The reader who wishes to try his hand at constructing curves with numbers obtained by the mixing together of sexes or races will easily convince himself of the irregularity of the graphs thus obtained.

I shall not emphasize any further the advantages that the construction of these curves renders. The reader will see, in the course of reading this Paper, how their study has permitted me to determine certain causes of the variations of skull volume as well
as to uncover existing relations between quite variable values; at the same time these curves present very few apparent relations, such as the circumference of the skull versus its volume, that researchers frequently mistake for actual relations. The simple aspect of these curves reveals immediately relations which, prior to these curves' construction, had been extremely difficult to discover. It was thus, for example, that, after having superimposed the curves of the distribution by age of the population in France and Italy, and then ascertaining that these curves only differed noticeably at their extremities, I immediately saw that these differences originate from the fact that there are in Italy more young people and above all more children than in France, but fewer older persons. Based on my studies, it is evident that if statistical documents ever entirely merit one's confidence, it will clearly be a most curious fact, indeed, an exception to the rule.

In cases where construction of the curves did not furnish a meaningful interpretation of phenomena, I made recourse to other processes which the reader will find examples of in this Paper. These processes are much too simple and uncomplex to necessitate an explanation. ${ }^{1}$

## FOOTNOTE

1. Most of the curves presented in this Paper have appeared in the section of the Paris Universal Exposition devoted to the anthropological sciences; next to these curves were statistical tables compiled by diverse authors, as well as my own clear explanations of these curves, which allowed even those possessing the most ordinary intelligence to understand their construction and purpose. It was not therefore without amazement that I read in an important Revue an article whose author, a theologian named Didon, talked in general about the various statistical works appearing in the Exposition, and about mine in particular in a way that betrayed uncommon levity and a singular ignorance on his part.

## II. Instruments Employed

Usefulness of taking measurements on the living. - Inadequacy of anthropology instruments for portable use purposes. - Description of a compass of coordinates that allows one to measure on the living the diverse angles and diameters of the skull and to reconstitute the person's profile. Different methods of measurement. - Cubage. - Necessity of following precise rules. - Examples showing the impossibility of making use of the cubages originating from foreign laboratories.

The instruments used today in anthropology laboratories are in general very accurate, but they present the double inconvenience of being not very portable and of having been constructed much more for the mission of taking measurements on the skull than on the living. As a result, nothing is more rare than to see a scientist who has travelled to a foreign land bring back some adequate measurements of races visited by him. Additionally, nearly all of the craniometric measurements that we currently possess have been taken on skulls and not on living heads. It should therefore be easy to understand that it will not be until we possess a great number of measurements taken on living subjects, whose aptitude and intelligence are known, that we will be able to plainly establish the relations existing between the shape and capacity of the head and intelligence.

The first part of this Paper being devoted to the study of the variations of skull volume, I shall not spend a long time on the instruments employed for measuring these variations and will instead simply restrict myself to providing a description of: 1) a new apparatus which permits one to obtain the exact profile of the head, and 2) the method that I use for comparing the profiles obtained to one another.

The method of the comparison of heads is very simple and allows one at a glance to appreciate differences that normally call for columns of numbers in order for such differences to be expressed. In employing this method, the profiles of various heads are first obtained on tracing paper and then are compared by mere superimposition; this permits one to immediately apprehend their analogies and their differences. In fact, nothing is easier and at the
same time more precise. One who uses this method immediately recognizes its superiority to the traditional approach that restricts itself to the comparison of groups of numerical measurements.

The only difficulty with the above method consists in the obtaining of very exact profiles of the head and face. As there hasn't been any instrument available that permits one to obtain a rigorously exact profile without the subject remaining immobile for several seconds, I decided to invent a special apparatus which I shall now describe. It realizes, I believe, these two significantly important considerations: it is portable and easy to use. In fact, it can be carried in a small case, and its handling can be learned in a few moments. Additionally, its price is fairly minimal.

This instrument, which I've named the pocket cephalometer or compass of coordinates, has been manufactured according to my designs by M. A. Molteni. It is based on the principles of geometry that permit one to determine the position of a series of points in space by means of their coordinates. Through its use one will discover that the pocket cephalometer allows one to immediately obtain all the diameters, curves and cephalic angles of the head that one is in the habit of measuring. I shall now describe this instrument and its usage.


Figure 1. The pocket cephalometer

The pocket cephalometer (see Figure 1) is composed of a 25 centimeters long steel ruler (AB), detachable at its center in a way so that it is able to be separated into two parts. Upon this ruler slide fairly friction-free two vertical metallic rulers (AC and BD) of 12 centimeters height, susceptible to being immobilized by the tightening of each's screw, and a small graduated ruler (EF), possessing scale marks along its entire length; one can likewise immobilize ruler EF at a pinch. Rulers AC and BD only move in lateral directions; they are terminated at their upper part by movable points C, D like those of a compass and which leave, when one removes them, grooves through which points C and D can be set by the pressure of a threaded screw. Small ruler EF, wholly possessing the lateral movement of the two others, and remaining like them perpendicular to the large ruler AB , is in addition endowed with top-to-bottom and bottom-to-top movements which give to it the possibility of sliding itself above the level of ruler AB. This double movement quality allows small ruler EF to follow the contours of a curve while always remaining constantly perpendicular to large ruler $A B$ upon which it places itself. If, for example, one sets with the threaded screw the two vertical rulers about the end points of the diameter of a cylinder, one would be able with the small ruler to pass across and carefully examine the entire half-circumference of this cylinder, with the small ruler being obliged to remain constantly in contact with the cylinder during this process. Because small ruler EF is graduated, it's sufficient to observe how many millimeters it has sunk down below or risen above the large ruler, assuming it has surveyed a given space, in order to obtain the ordinates of the different points of the curve, elements which will allow for the reconstructing of the shape on paper by very simple graphical methods.

Despite its simplicity, this process will take too long in actual practice; and, as I shall soon demonstrate, I have had no recourse but to complete the data-collecting by much more rapid means.

## USAGE OF THE POCKET CEPHALOMETER

As a general observation, I must point out that the small ruler's scale should be set even with and pressed lightly against the points
where the rulers are coincident with the sought-after divisions of the head (without taking into consideration projecting or receding parts that these divisions can contain and which, however, the design of the instrument compensates for). It is only necessary to take care to set on the left side vertical ruler AC, which must be stationed on this side, and which to that end is indicated in Figure 1 by the letter G. As for the small ruler, it must be positioned in a way such that its pressure screw lies in front of the observer. Once the rulers are thus arranged, and the apparatus fixed in place, one can accurately determine the three main head measures on the same plane. Here now are ways in which the pocket cephalometer can be of service.

Diameters of the Head. - The principal diameters that it is useful to measure on the head are, as one knows, the transverse and antero-posterior diameters and the vertical diameter. Because the first two can be calculated as easily upon the head as on the skull, one only needs to be careful in removing the points of the instrument whenever one operates upon the living. The vertical diameter is measured on a skeleton from the occipital foramen to the top of the skull; but, because the occipital foramen is not accessible on the living, one should take as a reference mark either the ear cavity or the top of the tragus's convexity. I believe that one can obtain an even more certain reference mark by choosing the lower rim of the middle partition of the nose, which pretty much corresponds from my observations to being even with the lower margin of the occipital foramen.

In order to measure the antero-posterior and transverse diameters of the head with the instrument, one must first lay the left branch (ruler AC), whose bottom edge must coincide with the zero mark of the horizontal ruler, on one of the end points of the diameter being measured, and then slide the other branch (ruler BD) until it reaches the other end point of the diameter. One now renders the instrument immobile with the pressure screw, and it only remains for one to read at the level reached by ruler BD's lower edge the number of millimeters indicated by the large ruler.

In order to obtain the vertical diameter, one first removes the right branch (ruler BD) and then places the left branch upon the top of the head. The large ruler being now in a quite vertical position, which with its upper knob a plumb line can be easily established, one next brings the small ruler even with the reference mark, that is to say, even with the ear cavity or, as I have recommended and as is indicated in Figure 2, even with the bottom rim of the nose. With the small ruler being held in place by the pressure screw, there is nothing more to do but to read on the large ruler the number of millimeters indicated: this number represents the vertical height sought.

I have to remark that this instrument is the only one, outside of the bulky instruments in laboratories that can only collect data there, which enables the vertical height of the head to be measured. It might seem at first sight that this measurement can be effectuated with calipers; but, it is sufficient to reflect but for an instant in order to realize that this idea is quite impossible. In fact, it is evident that the top of the skull and the ear cavity, where the lower part of the nose acts as a reference mark, are not in the same plane; their distance measured by calipers is an oblique linesuch as FH (in Figure 2), hypotenuse of right triangle FHO whose vertical side HO represents the height of the skull, a height that can clearly be determined because length $A D=H O$, and $A D$ is produced and shown by my instrument.


Figure 2

The height of the head substantially influences its volume, and in anthropology this fact is of considerable importance. It is obvious that the volume of a solid substance, such as a cylinder for example, can only be evaluated by knowing its height. It is simply the difficulty of measuring this height upon the living that has caused it to be neglected up till now. Although my instrument is mainly intended to collect measurements upon the living, I should point out that one can employ it on the skull with as much ease as if one used calipers. By reason of the length and height of its sections the pocket cephalometer can, in fact, attain all the points that are accessible to calipers. It accommodates, for example, being inclined a little to the right or the left in the taking of the diameter proceeding from the lower edge of the occipital foramen to the nasofrontal seam, a diameter which appears at first sight, because of the projection of the nose bone, unattainable for an instrument possessing parallel sections.

Measurement of Cephalic Angles. - The most commonly used of the cephalic angles is the facial angle of Camper. It is determined, as one is aware, by the intersection of two lines, one being horizontal, running from the ear cavity to the lower part of the nostrils, the other being more or less inclined upon the first, passing through the most projecting point on the forehead and the lower portion of the nose's dividing membrane. With various observers this latter reference mark is replaced either by the lower edge of the alveolar ridge or by the part of the face which juts out the most.

As I shall soon proceed to explain, because the pocket cephalometer enables one to obtain an exact profile of the head, nothing is simpler than to measure upon this profile with a protractor, like one does on the profiles of skulls sketched in craniography, the facial angle and the various cephalic angles having the ear cavity for a reference mark, such as the auricular angle, for example.

If one desires to limit himself to determining the facial angle of a person without bothering to obtain his profile, this can be quite easily accomplished by the following operation. It will be sufficient,
given the instrument's precise capability of measuring the vertical height of the head, to measure the distances existing: 1) between the ear cavity and the lower portion of the nose's dividing membrane, 2) between the ear cavity and the most projecting part of the forehead, and finally 3) between this latter point and the lower part of the nostrils. These three lengths represent the sides of a triangle that one can then construct and draw on paper by elementary geometric methods, and upon which one may measure the angle sought.

For the reason already mentioned above, it will once again prove quite impossible, as one might propose in error, to determine with calipers the lengths destined to serve as the basis of the preceding triangle. The facial angle, in fact, is contained in the middle plane of the head-an inaccessible plane-but still my instrument supplies the projection. The distances that separate the forehead and nose from the ear, taken with ordinary calipers, represent the sides of an oblique plane-sides naturally longer than those obtained in the vertical plane.

## Measurement of the Profile and Circumference of the Head.

 One may be able with this cephalometer to construct and draw point-by-point the profile of any region of the head, by trying to find, with the help of the small movable ruler and the two other stationary ones, the ordinates of each of these points; but, this approach is slow and tedious. I resort to other means, as I have previously stated, to determine the various points of the face that I wish to know exactly.In order to obtain an exact profile of the head and face, I operate in the following manner: having retracted the small ruler and the two points that terminate the extremities of the two branches (rulers AC and BD) of the instrument, and opening as widely apart as possible the latter, the subject's head is inserted perpendicularly inside the restraint that terminates the right branch; I then set with the pressure screw a think lead strip of 1 millimeter thickness, 5 millimeters width, and 60 centimeters length upon this branch's lower extremity. Next, with the right hand applying the extremity of this branch upon the nape of the
neck at a point even with the external occipital protuberance, one can now follow and pursue with the lead strip held in the left hand all the contours of the head as far as the top of the nose. Having arrived at this point, one exercises caution by making sure to pass the strip over the face in such a way that it does not squish down the soft parts that are unable to resist its pressure; exercising such caution, one continues on a little farther, bringing the strip under the chin as far as the neckline. The strip's end is then set by means of the second branch of the compass, producing a suitable end point. With the branches of the instrument being now immobilized in a definitive fashion, I withdraw the cephalometer from the head, and the lead strip fastened at the two branches' extremities in an invariable way is now placed upon a sheet of paper. It only remains to follow with a pencil the circuit created by the lower edge of the strip in order to accurately obtain, minus the details of the face (which one can complete, as I shall soon reveal), the contour of the head. One can assure himself by repeating this operation several times that the lead strip perfectly preserves the contours upon which it is molded.

The entire circumference of the skull and its transverse curve can be taken in the same way with a thin lead strip. The curved line formed by the latter is held in place at the two end points of its diameter by the branches of the compass.

The only inconvenience the lead strips present are their darkening the skin a little bit and their sometimes snapping and breaking, which can happen if one does not take the precaution of passing them through the flame of a gas lamp in order to anneal them whenever they have been in use for some time. I have conducted research to see if they could be replaced, but I have not found anything quite as satisfactory. Iron wire of 1 millimeter thickness covered with silk, like the kind employed to conduct electrical currents, and pure silver wire having a diameter of $1 / 3$ millimeter are, after the strips of lead, what seem to me best to use.

Profile of the Face. - By means of the preceding operation one obtains the exact contour of the head, but not that of the face. Should one have an interest in obtaining the latter, which is useful
notably in precisely determining the degree of prognathism (more or less great) of the various races, I suggest one proceed in the following way, which is to apply the previously expounded principles of analytic geometry. Let us suppose that the head is secured between the two branches of the instrument in the manner indicated by Figure 3. It is evident that learning the ordinates of the main points of the different parts of the curve included between $a a^{\prime}$ and $b b^{\prime}$ as well as their reciprocal distances from one another will permit one to discover the points through which this profile must pass. Nothing is easier than to obtain with small ruler RR' the length of lines $a a^{\prime}$, etc. One acquires these lengths by merely touching with small ruler RR' each of the projecting points of the face (a dozen is enough), and, when this ruler is even with each of them, you can then dictate to an aide the two numbers indicated, one being how much the point is elevated above the large ruler, the other being at what distance from zero it finds itself on this same ruler. A piece of paper, preferably graph paper, is now readied. For there is nothing more to do but to set up on a horizontal line a series of parallel lines showing in millimeters their lengths and respective intervals as expressed by the preceding numbers, in order to obtain with the most rigorous precision the points through which must pass the curve that constitutes the profile of the face; by carrying out this procedure, the profile of the entire head, already largely obtained, is now complete.


Figure 3

Finally, in order to have in place all the elements necessary for anthropometric research, one needs to mark upon the drawing the position of the ear cavity at the point where two circular arcs traced with a compass intersect each other, arcs whose radii have for spacing the length measured with the cephalometer from the ear cavity to any two points of the face.

I must add that in order to avoid the errors that one can easily commit with the previously-described graduated lead strips, I have used in my research on the inequality of corresponding regions of skulls a very simple and yet most exact measurement process. It entails determining whether each bone of the head that comes in a pair is equally developed on both sides, or whether each half of a head bone that does not make up a pair is equally developed. With a band of paper 1 centimeter wide by about 60 centimeters long, I place it horizontally around the head in such a way as to make the band cross behind on the external occipital protuberance and in front on the median line, above the brow ridges. With a pencil I will mark on this band the points where it comes into contact with the median line in front and back and with the lateral sutures. In measuring the different lengths contained between these marks, or more simply in folding the band upon itself so as to compare the opposite sides, I will see one after another which are the unequally developed sides of the head.

As for determining the cubic capacity of the cranium, this is a delicate undertaking that can only be conveniently executed by following the precise guidelines detailed in a special paper authored by Doctor Paul Broca, the founder and Secretary-General of the Anthropology Society of Paris. Cubages obtained by the old method are so inaccurate that the same cranium whose capacity is measured by two different observers or by the same observer retaking his own measurement often yields differences that can exceed 100 cubic centimeters. It is only by following the abovementioned precise guidelines that the total cubic capacity found for a cranium will be absolutely correct. Unfortunately, these guidelines are far from being sufficiently known by anthropologists. With foreign researchers each utilizing a different method, it results
that their measurements cannot be used or at least compared with those obtained by other researchers. As an example of this situation, I shall cite the case of a collection of 47 Finnish skulls that were sent by the Helsingfors Museum to the Universal Exposition. After having carefully taken the cubage with the aid of lead shot and according to Doctor Broca's rigorous precepts, it was discovered that not one of the indicated capacities for each skull in the brochure which accompanied them was in conformance. For many of these skulls the error amounted to 125 cubic centimeters. ${ }^{1}$

I have therefore felt obliged, under pain of removing any solid base to my conclusions, to almost exclusively support my calculations only on the measurements performed on skulls at the Paris anthropology laboratory (all the elements of which Doctor Broca kindly placed at my disposal). By neglecting nearly entirely the measurements taken by foreign researchers, I have naturally reduced somewhat my sources of information, but given similar material I believe quality of such seems much preferable to quantity. Moreover, I must remark that, when the number of skulls of a series attains a certain number, the advantage that might accrue by increasing this number with elements that one is not certain of is altogether offset by the chance of introducing errors. Additionally, given that the probability of the precision of a result does not increase with the number of observations, but rather with the square root of the number of these observations, it is necessary that in order to render the results two, three, or four times more precise, the observations must become respectively four, nine, or sixteen times more numerous, an amount which, however, will be impossible to perform given the relatively small number of museum inventory catalogs that presently exist.

## FOOTNOTE

1. I can cite many other examples that show the impossibility of comparing amongst each other measurements obtained by different researchers. The reader will easily understand that it was not without a thorough study of the foreign documents that I have been obliged to entirely renounce their use. As new proof of the impossibility of utilizing these documents, I shall mention the singular results that one will obtain by taking as a base the measurements of the capacity of numerous German skulls that have been recently published
in a special work by the learned anthropologist of Bonn, Doctor Hermann Schaaffhausen (Die anthropologische Sammlung des anatomischen Museums der Universität, Braunschweig, 1877). Doctor Schaaffhausen found the average capacity of 155 German skulls, originating from normal adult male subjects, to be only 1422 cubic centimeters, that is to say, less than the average skull capacity for Negroes. This result is too much at variance with the measurements effectuated by all other researchers on German skulls to be admitted for a single instance. The series is nevertheless very homogeneous; in other words, all the capacities (comprised between 1025 and 1920 cubic centimeters) increase, except at the most extreme points, in a progressive fashion. Now, the average of the capacity provided by my graphical method turns out to be exactly the same as that found by calculation. It is therefore probable that all of Doctor Schaaffhausen's measurements are attributable to some consistently applied error, most likely resulting from his use of an inexact operating method. The error is about $1 / 10^{\text {th }}$ of the volume found for each skull. So, although one might draw some interesting information from a study of this series of skulls, one must keep in mind not to compare it to any other.


Doctor Hermann Schaaffhausen

## III. Variations of Skull Volume and of the Weight of the Brain in the Human Races

Variations of skull volume in diverse races and in individuals of the same race. - They are much greater than what the averages indicate. - What might produce these variations. - How easily their size escapes the eye. - Limit of the variations that the radius of a sphere whose volume is equal to that of the largest and smallest human skulls might present.

The variations of skull volume that one observes, be it with respect to individuals of the same race or with respect to those of different races, are considerable. For example, among present-day Parisians of the masculine sex, the volume of the skull measures anywhere from 1300 to 1900 cubic centimeters, a range of 600 cubic centimeters. This difference increases to 800 cubic centimeters if one mixes in skulls belonging to present-day Parisians of the feminine sex. In bringing together the diverse human races and classifying their cranial capacities by increasing volumes, one can say, even after omitting the exceptional cases constituting the extreme points of the series, that among men the volume of the skull can vary from a little to almost twice as much. I shall restrict myself for the moment to this succinct observation, as I shall treat at length in a following chapter these differences that appear in subjects of the same race as well as in those of different races.

Before the employment of precise methods of measurement of modern anthropology, these differences had escaped the detection of most anatomists. "I have observed," stated Bichat, speaking of the diameters of skulls, "that the increase in one diameter only happens at the expense of two others, so that, notwithstanding the numerous changes to which the skull may be exposed, its general capacity will not experience differences as great as it appears at first."

In order to understand how the enormous variations in the volume of the head that I have pointed out had been able to escape the detection of such an illustrious anatomist, one needs to recall that the head approximates by its shape a spheroid and, given that the volumes of spheres are proportional to the cubes of their
radii, a very small increase in the lengths of these radii produces a very large increase in these spheres' volumes.

As an example of the above point, we shall compare spheres almost equivalent in volume to the largest and smallest skulls and then investigate how their radii differ. Given, let's say, a sphere 1251 cubic centimeters in size and another containing 1867 cubic centimeters, both of which are nearly equivalent to the smallest and largest heads, let us try to find out how these two spheres, of which one is $1 / 3^{\text {rd }}$ greater than the other in volume, differ by their radii. The formula

$$
R=\sqrt[3]{\frac{V \times 3}{4 \pi}}
$$

shows that the radius of the 1251 cubic centimeter sphere will be 66.8 millimeters and that the radius of the 1867 cubic centimeter sphere will be 76.4 millimeters. These two spheres, though displaying an enormous difference in volume of more than 600 cubic centimeters, only present a difference of about 1 centimeter in radius. Therefore if the human head was perfectly spherical, we would be able to say that between the largest and smallest human head the differences in the radius amounts to only 1 centimeter.

This very simple calculation shows us why the diameters of the head might vary only within very small limits, whereas its volume on the contrary might vary in very large percentages. I have calculated that on a sphere 1558 cubic centimeters in capacity, which is to say one that nearly corresponds to the average skull volume, a mere increase of 1.6 millimeters in the radius produces an increase in the volume of over 100 cubic centimeters.

One can now understand how such minimal differences in diameters had been overlooked by anatomists up to the present time where anthropologists have finally had recourse to precise methods of measurement. But additionally, the general habit of only publishing the averages has resulted in rendering little perceptible the differences in skull volume that exist between the races and above all between individuals of the same race. The
voluminous skulls that a race contains, being in effect always few in number, are without noticeable influence on these averages.

However much the differences in skull volume easily escape the eye, the same cause of error ought not to subsist when one weighs the brain, and since this operation has been practiced for a long time, it seems that we ought to have known for a long time the variations of weight that the brain undergoes. However, this is not at all the case. The published weights up to now are so insufficient that we see our most eminent anatomists commit large mistakes with regard to the weight of the brain. In the latest edition of his excellent work on anatomy, one of our most learned anatomists, Professor Sappey, having found as a maximum of the brains weighed by him the weight of 1510 grams, asserted that the brain that attained this weight was an "encephalon of an entirely exceptional size." Now, this weight is, in fact, so unexceptional that out of 100 individuals selected at random from patients at hospitals, one finds a dozen whose brain weight is equal to or greater than 1510 grams; this can be seen in the following table created from the weighings effectuated by Doctor Broca in hospitals, the various totals of which are contained in unedited materials placed at my disposition by the eminent professor.

## TABLE I

Variations of the Weight of the Brain among Present-day Parisians

## Weight of the Brains




Plate II. - Variations of brain weight, skull volume and the circumference of the skull in the Parisian population.

Above each curve (from top to bottom) is indicated what is representedbrain weight, skull volume, and skull circumference. The scale on the left side is the scale of weights ranging from 900 to 1700 grams. The two scales on the right side represent 1) skull volumes of 1300 to 1900 centimeters, and 2) the circumference in centimeters of skulls, from 49 to 56 centimeters.

With the spacing of the ordinates being proportional in our curves to the variations expressed in hundredths of the observed phenomena, it is sufficient, in order to know how many there are out of 100 brains of a given weight, and out of 100 skulls how many there are of a given circumference or volume, to look for how many millimeters vertically separate the points where the curve intersects the horizontal lines corresponding to a given point on the scale. For example, let's say we wish to investigate how many brains out of 100 weigh from 1500 to 1600 grams. The number of millimeters being 10 (determined by counting a total of 10 horizontal lines from the level of 1500 to 1600 grams that intersect the vertical displacement of these two endpoints on the curve), we see that there are 10 individuals out of 100 whose brain weight varies from 1500 to 1600 grams.

From the preceding we are therefore able to conclude that, although unrecognized by most anatomists, the differences in skull capacity and brain weight existing not only between the human races, but also between individuals of the same race, are extremely considerable. We shall now investigate the causes of these differences.


Most present-day anatomists, including the distinguished Professor Constant Sappey (above), have failed to recognize that the differences in brain weight that exist between the human races and also between individuals of the same race are extremely considerable.

## IV. Research of the Causes of the Variations of Skull Volume and the Weight of the Brain

Rarity of works on this question. - Inadequacy of the methods employed. § 1. Influence of sex on the weight of the brain. - Table showing that given an equal height and weight the female possesses a brain significantly less heavy than that of the male. - Table showing that men and women tend to differentiate themselves more and more by their brain, and that this differentiation is so much the greater in the most elevated races. - Small capacity of the female skulls in the superior races. - The races where the male skulls occupy the highest levels of development are often the ones where the female skulls occupy the lowest levels. - § 2. Influence of height on the weight of the brain. Contradictory opinions expressed on its influence. - Study of 100 brains whose weight is known. - Necessity for a thorough analysis of these numbers. - Limited influence of height. - § 3. Influence of body weight on the weight of the brain. Complete absence of data on this question. - Research allowing one to appraise the importance of its influence. § 4. Influence of age on the weight of the brain. Increasing weight of the brain up to a certain age and then its diminution. § 5. Influence of race and level of civilization on the weight of the brain. - Table permitting the appraisal of this influence. - Inadequacy of the results provided by averages. - The superiority of a race over another is constituted by the number of voluminous skulls that this race contains. - Inferior races are unable to exceed a certain cerebral capacity limit. - Table showing that as a race becomes more civilized, the individuals who compose it tend to more and more differentiate themselves by their brain volume. - § 6. Relation between brain volume and intelligence level. - Comparison of the skull volume among 1200 present-day Parisians belonging to different social categories. - What, from the psychological point of view, superior intelligence is consisted of. - How much influence does education have on the development of the brain?

The number of authors who occupy themselves with the causes accounting for the variations of skull volume and brain weight is very limited, and one must go back to Parchappe, who wrote 40 years ago, in order to obtain an entire work on this question. As Parchappe did not have at his disposal but a small number of measurements, he was unable to arrive at precise results, as on more than one occasion our conclusions turn out to be completely opposite to his. Moreover, in spite of the works of Parchappe and various other authors on the causes affecting the weight of the brain, the largest incertitude regarding this matter still prevailed among the anatomists. Some, such as the author we have cited, believed that the person's height has a considerable
influence on brain weight; others, such as Cruveilhier and Bichat, maintained, on the contrary, that it has absolutely no influence. What I shall say about the influence of any random factor might be said of all the others, the influence of the development of the brain on the state of intelligence, for example.

As it is solely a matter here of observable facts, and also that these facts cannot vary from one observer to another, it is only to the imperfection of the interpretive methods used that the divergence of the obtained results can be attributed. Let us therefore investigate where to take up these methods and whether it will be possible, by utilizing more certain means of analysis, to cast some light on the various factors that can affect skull volume and consequently the weight of the brain, the development of one of these two elements being in the immense majority of cases in direct relationship with the development of the other.

## § 1. Influence of sex on the weight of the brain

Although nothing is easier, when studying a sufficiently numerous series, to ascertain than whether or not the brain of men is heavier than that of women, the opinion of anatomists is less than unanimous on this question. Their uncertainty is readily explainable when one notes that there exists in each series, as one can see by the aspect of our curves, a small number of female skulls whose capacity is more voluminous than the average volume of the male skulls. What can result when one restricts himself to a few measurements is that one might by happenstance light upon just these exceptional brains, and from these albeit correct measurements draw very false conclusions. I am therefore not astonished by the opinion of the anatomist Sœmmering, who believes that the head of the female is larger than that of the male, or by the opinion of Bichat, who thought that one's sex has little influence on skull volume. It is in this manner that Milne-Edwards, in his Leçons sur la physiologie et l'anatomie (1876), asserts that "considered in an absolute way, the brain of a man is larger than that of a woman, but in proportion to the body's mass the difference in sense is the reverse."

After weighing thirty female skulls and other male skulls, Parchappe concluded that the weight of the second was greater than that of the first; but, as he did not conduct his research on subjects of the same height, one can wonder-and he himself, moreover, posed this objection without being able to resolve it-if this absolute inferiority is not able to be, as Milne-Edwards allows, compensated by a relative superiority, that is to say, whether given equal height between the two sexes the female brains will not be as heavy or even heavier than those of the males.

In order to answer this question, I have researched among the hundreds of brains weighed by Doctor Broca male and female subjects of the same height and compared the average of the weight of their brain. This comparison, which was carried out on 17 brains of each sex, has yielded the following results:

## TABLE II

Average Weight of the Brain in Men and Women of Equal Height

## Average brain weight of men between 154 and 163 centimeters in height . <br> 1322 <br> Average brain weight of women also between 154 and 163 centimeters in height . <br> Difference in favor of the male brains

This comparison clearly proves that it is not height which determines the difference of brain weight that we observe between the two sexes: this difference is therefore inherent to sex itself.

One might perhaps object to the preceding findings establishing brain weight differences favoring men by noting that, given equal height, women generally weigh less than men; but, to this objection I can easily respond by presenting new results demonstrating that by focusing less on height and more on weight, women of equal bodily weight have a brain much less heavy than that of men. For example, I have surveyed a thesis by Doctor Pierre Budin (De la
tète du foetus, Paris, 1876) which contains the measurement of the circumference of the brain of 49 newborn girls and boys of equal weight, and I was able to obtain, after assembling suitable groupings, the following table:

## TABLE III

Comparison of the Increase in Circumference of the Head with Bodily Weight among Newborn Males and Females

# Weight of the Newborns 

From 2500 to 3000 grams
From 3000 to 3500 grams
From 3500 to 4000 grams

Circumference of the Heads
Males
38.0
38.8
40.1

Females
36.7
38.2
38.7

The circumference of skulls is, as I shall show in another part of this Paper, in straight relation with their volumes when we deal with a series of skulls; and now we see, in the preceding table, that when comparing male and female individuals of the same weight, we obtain the same aforementioned result, that is to say, that the skull volume of the female is very significantly less than that of the male.

In all the races of mankind, the skull of the female is less voluminous than that of the male, but the degree of inferiority varies considerably from one race to another. Studying the graphs that we have constructed and the associated numerical totals that have allowed for their construction leads us to recognize this most curious fact: the differences which exist between male and female skulls of the same race constantly increase as one goes up from the inferior races to the superior races, so that, from the point of view of the mass of the brain, women tend to differentiate more and more from men. By examining the following table one can easily see the progression of these differences.

## TABLE IV

Comparison of the Average Capacity of the Male and Female Skulls in the Inferior and Superior Races

Average Volume
of Male Skulls

Average Volume
of Female Skulls

Difference between the Average Capacity of the Skulls of the two Sexes

## RACES

| Pariahs of India | $\mathbf{1 3 3 2}$ | $\mathbf{1 2 4 1}$ | $\mathbf{9 1}$ |
| :--- | ---: | ---: | ---: |
| Aborigines of Australia | $\mathbf{1 3 3 8}$ | $\mathbf{1 2 3 1}$ | $\mathbf{1 0 7}$ |
| Polynesians | $\mathbf{1 5 0 0}$ | $\mathbf{1 3 8 1}$ | $\mathbf{1 1 9}$ |
| Ancient Egyptians | $\mathbf{1 5 0 0}$ | $\mathbf{1 3 6 3}$ | $\mathbf{1 6 5}$ |
| Merovingians | $\mathbf{1 5 3 7}$ | $\mathbf{2 2 2}$ |  |

In the interest of clarity and brevity, I have chosen not to greatly expand this table by listing all the numbers which justify the fact that as we elevate ourselves up the ladder of civilization, men differ more and more from women. All the skulls studied up to now, including those I have not cited, such as skulls from China, Africa, the Stone Age, etc., demonstrate this progressive differentiation. There exist races, like the African Negroes, for example, of which we possess numerous skulls, where it is completely impossible to attribute to chance the minimal difference observed between male and female skulls of the inferior peoples.

Although they vary from moderately to over twice as much, the differences that the preceding table presents still understate reality, because keep in mind that they emanate from the averages which, as I have said, efface all the extreme cases. In order to demonstrate how these differences are in reality vast, it is sufficient to recall that the most inferior human agglomerations listed in Table IV (Pariahs and Australian aborigines) possess an average male skull capacity of about 1332 cubic centimeters, and the superior agglomerations (present-day Parisians) possess a skull capacity of around 1559 cubic centimeters, which represents a difference of only 227 cubic centimeters; hence, the difference that exists between the average skull capacity of the most and least
elevated human races is barely equal to that existing between the average skull capacity of male and female Parisians. ${ }^{1}$

This progressive differentiation we see taking place of men from women as we raise ourselves up the scale of races and, consequently, civilization can hardly be of any surprise to us. In the inferior races the superiority of men over women is very inconsiderable; in such races the woman participates with the man in his labors, often even working more than him, and necessity renders her industrious. In the completely civilized races, the Latin nations notably, the woman leads a quite different life from that of the man. The education that she received does little to improve her intelligence, and tends rather to restrain it than to develop it. Her intelligence therefore remains stationary or decreases whereas, with the man instructing and improving himself more and more with each generation, the progress accumulating by heredity ends up by gradually distancing farther away the woman who intellectually differed very little at first from the man.

The preceding observations ought to lead to the following result: the races where the average capacity of the male skulls is the greatest should not at all be those where the female skulls are the most developed. Investigating the average skull capacity of diverse human races has permitted me to immediately verify the accuracy of this theoretical concept. By carefully categorizing in separate fashion first the male skulls and then the female skulls, instead of mixing both together as so many authors still do, one arrives at this curious result: the races where the male skulls occupy the highest level are often exactly those where the female skulls occupy the lowest. The following table, in which the races are categorized by average cranial volume, clearly illustrates this fact.


Male Australian aborigine;
average skull capacity $=1338$ cubic centimeters

## TABLE V

# Average Capacity of Female Skulls of Different Races 

# Skull Capacity (cubic centimeters) 

Female skulls found in the Baye grotto (dating from the Stone Age)
Skulls of Merovingians
(environs of Paris, about the $7^{\text {th }}$ Century)
1407

Skulls of Polynesians
1383
Skulls of Parisians 1381

Skulls of New Caledonians 1337

Skulls of Negresses 1330
Skulls of Negresses . . . . . . 1252
The same categorization repeated on skulls of the same races, but belonging to the male sex, gives most different results. For example, Parisians occupy in this case the top rank by the volume of their skull. We thus witness this seemingly strange fact of a race whose male subjects occupy the most elevated rank by the volume of their brain, but whose female subjects occupy by contrast one of the lowest ranks. The skull volume of female Parisians, in fact, ranges from that of the female skulls of the inferior races where, obliged to share in the work of man, the woman is compelled to frequently exercise her aptitudes.

Reproducing some of the numbers of this Paper, published in part in reports produced by the Academy of Sciences, a distinguished scholar, M. G. Pouchet, Assistant Professor of Physiology at the Sorbonne, has recommended their review to supporters of equal rights between men and women. Above all, I myself recommend these numbers be reviewed and their meaning understood by educators.


African Negress; average skull capacity = 1252 cubic centimeters


## Plate III. - Skull volume of women of diverse race compared to the skull volume of men of the highest and lowest races.

The scale ranges from 1050 to 1900 cubic centimeters. These curves show: 1) the relations existing between the skull volume of men and that of women;
2) that although the difference in skull volume between men of the superior and inferior races is very large, the difference between women of diverse races is quite small; 3) that the female skull of civilized people is much nearer in size to that of men of the inferior races* than to that of men of the superior races; 4) that women of the superior races do not at all occupy the rank occupied by male individuals of the same races. The explanations which accompany the other plates (most particularly Plate V) provide all the directions for the reading of these curves.

* The curve of the skull of the inferior races has been constructed by combining the male skulls of all the lowest races-Australian aborigines, Bushmen, Hottentots, etc.-that the Museum of Anthropology contains.

By attentively examining our curves, one sees that there are in each race a certain number of female brains larger in capacity than a certain number of male brains of the same race. This very limited number of such brains will not modify at all our preceding conclusions. It merely shows that in each race there are a small number of women whose brains have attained such a size. It is with difficulty, moreover, that the most voluminous female skulls surpass by a little the average of the male skulls (that is to say, the simple mediocrity) and also quite problematic that the psychological side will correspond exactly to the anatomical side.

I ought not to omit remarking that, when one operates on the averages, the differences that one obtains in comparing the skull of men and women are even lower than those which appear when one compares-which our curves permit us to do-the largest female brains to the largest male brains as well as the smallest female brains to the smallest male brains. Studying these same curves will also show this theoretical fact, which is evident besides, that women differ much less between themselves than do men by the capacity of their brain. One will also notice that in the most intelligent races, such as present-day Parisians, there is a considerable percentage of the female population whose skulls come nearer in volume to the skulls of gorillas than to the skulls of the most developed males.

This is not the place, in a Paper solely dedicated to the demonstration of anatomical facts, to investigate whether the inferiority of the female skull, principally in the higher races, is accompanied by a corresponding intellectual inferiority. I shall therefore restrict myself to responding to this question in a few words. This inferiority is too evident to be contested for an instant, and one can hardly discuss the matter but as to its degree. All psychologists who have studied the intelligence of women acknowledge today that, except for the poets or romance writers, they represent the lowest forms of human evolution and are much nearer to savages and children than to the civilized adult male. They are primarily characterized by instability, fickleness, absence of reflection and logic, incapacity to reason or giving way to improvident reasons, and the propensity of having only the instinct
of the moment as their guide. Additionally, one will not be able to cite in the sciences which require reasoning a single remarkable work produced by a woman, and yet a great many have received a scientific education. In America alone 600 practice medicine. It is only in certain arts in which women exert themselves in an unconscious way, like music, singing, poetry, etc., areas where perhaps primitive peoples and savages excel, that one observes them at very rare intervals distinguish themselves.

What maybe constitutes to a woman a substantial advantage over a man is the possession of a quite certain instinct which allows her to unconsciously divine things that the latter frequently cannot see but in a way confused by reasoning. It is a precious aptitude, but one possessed also by the majority of inferior creatures. It is in the same category as the instinct which tells the ape whether the food that he holds in his hand will be useful or harmful to him, or tells the bee what is, among the numerous shapes that he can give to each cell in the honeycomb, the one that will contain the most space with the least expenditure of material possible.

One cannot deny that unquestionably there exist very distinguished women who are quite superior to average men, but such cases are as exceptional as the birth of some monstrosity such as, for example, the birth of a gorilla with two heads; in short, such cases are wholly negligible.

What is believed to make with respect to poets and novelists the woman superior over man, is uniquely-outside, of course, her uncontested physical qualities-the exaggeration of her sentiments. But this same exaggeration contributes even more than the inferiority of her intelligence to drawing her nearer to the level of savages and children, let alone even the lowest mammals. Maternal love, for example, is much more developed in certain monkeys, such as the guenon, than in the woman because the former never outlives her young ones (assuming they enjoy a fairly normal life span). Certain birds enter into indissoluble unions where they give proof of the most tender and faithful sentiments, and the love felt by the female for her companion is so deep that she soon dies of grief when death comes to carry him away.

One who invokes in favor of the intellectual development of the woman the considerable role that she plays in the march of human affairs and the fact that she often guides us by her wishes forgets that man is much more governed by sentiment than by reason, and that it is precisely because the woman acts exclusively on our feelings, which are the domain of her unconscious instinct, that she frequently has so much control over us. Outside of the motives elicited by sexual attraction, which in reality constitutes women's unique force, man often allows himself to be tyrannized by them by a sentiment falling in the same category as that which makes him obedient to the will of infants or young mammals (like puppies or kittens) when they are caressed and grateful.

Those who have proposed to provide women an education similar to the one received by men have proven how much they are ignorant of the fair sex's true nature. It will undoubtedly be desirable that women be given an education entirely different from the one that they presently receive, which increases by too much the distance that they are removed from us; but wishing to provide the two sexes, as America has begun to do, the same education, and consequently to offer them the same ends and aims, is a dangerous chimera which will only result in stripping the woman of her role, obliging her to enter into competition with man, and ridding her of all that forms her value, usefulness and charm. The day where, despising the lower occupations that nature has assigned to her, the woman decides to quit her home and venture forth to take part in our struggles, on this day a social revolution will begin where all that comprises the sacred bonds of the family will disappear, leading to a deleterious future which one can say has never been more deadly.

## § 2. Influence of height on the weight of the brain

The first idea that comes to mind when one studies the quite considerable variations that exist among the brains of a given group of individuals is that these variations simply owe themselves to the differences in height of the individuals. If we consult the anatomists on this point, we will only meet with contradictory assertions. Cruveilhier states on page 440 in his book Anatomie
(1871), "it results from a great number of facts that the volume and weight of the brain are independent of the height of individuals." What these facts are Cruveilhier does not, however, tell us. As for Sappey, he limited himself to reproducing the opinion of Parchappe, declaring, based on the measurements carried out by him on a small sample of 5 tall individuals and 5 short individuals, that the brain of tall subjects is $6 \%$ larger than that of short subjects. The brain weight of the 5 tall men was 1330 grams, that of the short men 1254 grams, a difference of 76 grams. I must remark that when one has experimentally established (as I have) how much a brain weight differs among individuals of the same height, one recognizes that measurements effectuated on such a small number of subjects are without value.

In order to wholly determine the influence that height exerts over the weight of the brain, one must possess a large number of weighings of brains of subjects whose height is known. As science has not heretofore provided this data, I do not know how it would have been possible to resolve this question had it not been for the graciousness of Doctor Broca who shared with me the unpublished material that he has assembled for 10 years in the hospitals of which he is the surgeon. The material consists of the weighings of the brains of several hundred individuals whose height has been taken. Here is the way I have utilized this valuable data. I took at random from the patient record books the brain weight of 100 individuals whose height was indicated, but then I classed the individuals by increasing brain weight, taking care not to omit listing alongside this amount each person's corresponding height. The following table and the curves that accompany it present the result of this classification.


Doctor Paul Broca

## TABLE VI

Brain Weight, Arranged by Increasing Progression, of 100 Individuals, with their Height

| Height <br> cm. | Brain <br> Weight <br> gr. | Height <br> cm. | Brain <br> Weight <br> gr. | Height <br> cm. | Brain <br> Weight <br> gr. | Height <br> cm. | Brain <br> Weight |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| gr. |  |  |  |  |  |  |  |

With the numbers from the above table I have constructed in Plate IV the curves of the heights and brain weights. One sees on Plate IV that exactly above points where the brain weight curve intersects any ordinate will be found the curve of the height intersecting the same ordinate, and the brain weight and height pertaining to the individual will be found on the same vertical line. Now, if the weight of the brain increases at the same time as the height, the two curves will be parallel or at least go up at the same
time; but, one can see from glancing at Plate IV that this is hardly ever the case. Moreover, the preceding table already shows that if height has an influence, this influence is well concealed, because we see very tall individuals possessing a much less heavy brain than many short individuals, and we also observe individuals of the same height having brains that differ in weight by several hundred grams.

From the preceding should one conclude that there is not, as Cruveilhier asserts, any sympathy of relation between height and brain weight? At first I clearly inclined towards this hypothesis, but not wishing to arrive at a similar conclusion without having more deeply analyzed the problem, I submitted the preceding numbers to a more heedful examination and was soon convinced that in spite of appearances there does exist, in fact, a real relation between height and brain weight.


The anatomist, Jean Cruveilhier (above), mistakenly believed that brain weight is independent of the height of the individual.


Plate IV. - Minimal influence of height on brain weight; limited nature of this influence in the groupings by series.

The $1^{\text {st }}$ scale on the left side represents brain weight in grams, from 900 to 1750 grams. 1 millimeter = 10 grams.

The $2^{\text {nd }}$ scale on the same side represents height in centimeters, from 145 to 185 centimeters. 1 millimeter $=1$ centimeter. The curve which regularly rises from 940 to 1725 grams is the curve of the weight of the brains arranged in an increasing way. The very irregular curve that cuts through a large number of points is the curve of the heights that the individual possessors of these brains have. Each vertical line contains, as one can see, two points, one indicating the weight of the brain, the other the corresponding height. It is the union of all these points by lines which constitutes the two curves.

As the appearance of these two curves does not seem to reveal any relation between brain weight and height, it is only in the groupings by series that one is able to discern the influence of the latter. The horizontal line which crosses obliquely the plate from 1289 to 1387 grams shows the limited nature of this influence (in fact, very small).

In order to prove that a real relation exists between height and brain weight, we shall group together all the brains of individuals of the same height and then calculate the average brain weight of each group. By doing this, we thus obtain the following numbers:

## TABLE VII

Influence of Height on Brain Weight in the Groupings by Series


The preceding table not only shows clearly that height has an influence, but also it shows none the less clearly that this influence is quite minimal, inasmuch as for each 10 centimeters increase in height the average brain weight barely increases but 40 grams, and additionally, between the average weight of the group of male brains of the shortest individuals and average weight of the brains of the tallest individuals the difference hardly comes to 100 grams. This amount is obviously rather trifling if one compares it to the differences of many hundreds of grams that one frequently encounters between individuals of the same height. Moreover, the mere fact that individuals of the same height possess very differently weighing brains proves that aside from height other more powerful factors act on and neutralize its influence. Height therefore is a factor which has an effect on brain weight, but it is a factor with slight importance whose influence will most often disappear in the face of other influences.

I must add that the 100 individuals whose height and brain weight I have compared represent quite well others in the general population. The curve of their height, which I have constructed in accordance with my method, deviates very little, in fact, from the one that I have also constructed with the numbers pertaining to the height of the French population (this data originating from official documents detailing military recruitment statistics).

## § 3. Influence of body weight on the weight of the brain

The rare authors who have entertained the causes which might make the weight of the brain vary have totally remained silent with respect to the influence of body weight. They have implicitly allowed that, this weight being in rapport with the height, the influence of one will be evinced whenever we sufficiently know the action of the other.

Unfortunately, there is not a constant rapport or proportionality between height and body weight. Sometimes short individuals weigh much more than tall subjects, and it is sufficient to examine Curves 3 and 4 of Plate I in order to convince oneself that even amongst individuals of the same age (newborns) there is no parallelism between the increase in weight and the increase in height. Far from diminishing after birth, the divergence only continues to enlarge afterwards. In fact, it is evident that a 1 centimeter increase in height will not produce the same augmentation of weight with a short person than with a tall person, for the same geometric reason that a 1 centimeter increase in the diameter of a sphere produces quite different enlargements in volume, depending on the diameter of this sphere. Moreover, even supposing that all the individuals being compared possess the same height and weight, an increase of 1 centimeter in height will produce in them an equivalent increase in weight only in the unlikely case where they will all be of the same age. Quételet's research has proven, in fact, that the weight of a child increases according to the cube of the child's height, but as the child approaches adolescence the increase in weight manifests itself based on an intermediate value (still undetermined) between the second and third power of the height.

The exact determination of the influence of body weight on the weight of the brain will therefore remain a very ticklish problem as long as, in order to resolve it, we do not have in hand valid measurements of the weight of a large number of individuals as well as accurate measurements of these individuals' cranial capacity or brain weight.

In the absence of this data which is presently entirely lacking in
science, we are nevertheless able to show by indirect means: first of all, that body weight has a considerable influence on brain weight, and then that, whatever the importance of this influence is, it is not the key factor to which we can attribute the brain weight differences that we observe among individuals of the same race.

In order to justify the first of these assertions-i.e., that body weight has a notable influence on the weight of the brain-one can invoke theoretical and experimental reasons, and these I shall now successively examine.

The brain, especially the mass of white fibers which constitutes a large part of its volume, does not have, as we know, for its unique functions just those which preside over intellectual phenomena. It is also a nervous center that is in more or less direct communication with all the organs, and one conceives that, according to the mass of the organs it maintains or the activity of these organs, its volume must be more or less considerable. It is for this reason that certain mammals weighing significantly more than man, such as the elephant and dolphin, also possess a brain weighing considerably more than man's.

Independently of these considerations, we can present experimental proofs in favor of the body weight/brain weight connection theory. The table (Table III) appearing earlier in this Paper, composed of the weight of around 50 newborns whose skull circumference was known, shows, in fact, that the average circumference of the head and, consequently, as we have already noted, the weight of the brain, increases with body weight in a rapid fashion.

Experimental proof from another category of research also happens to demonstrate the influence of body weight on brain weight. The reader who wishes to make an effort of copying Curve No. 2 of Plate VII which represents the diverse circumferences of the head of 1,000 present-day adult Parisians, and then superimpose it on Curve No. 4 of Plate I which represents the body weight of 1,000 newborns, will see that these two curves coincide in a remarkable way through the longest part of their course, which
certainly appears to demonstrate that differences in skull volume and, by consequence, brain weight are clearly in part the result of differences in body weight.

What are the outer limits of this influence? It is something which, as I have already said, cannot be determined yet due to lack of sufficient data; but, we do know enough already to rigorously prove that it is not at all the variations of body weight that produces the considerable differences of brain weight that one observes between male and female individuals, between individuals of the same sex or race, and lastly between individuals of different races.

As far as individuals of different sexes are concerned, we have provided earlier in this Paper quite evident proof of a correlation between brain weight and sex. We have seen that among female and male individuals of the same weight, height and age, there is a significant difference in skull volume in the favor of men and, consequently, one's sex has a considerable influence on brain weight.

With regard to individuals of the same sex and same race or of different races, the proof that body weight variations are not the principal factor responsible for the differences in brain weight is evidenced in a peremptory way by the simple aspect of our diverse curves or by the numbers that have served to construct them. The inferior races (like the Negroes, for example) certainly do not possess a body less heavy than that of the superior races, and yet we see (Plate V, for example) that their brain is constantly less voluminous and proportionally less voluminous as one descends the scale of races. Additionally, we see (Curve No. 1 of Plate VII) that in the same race skull volumes are very different according to the intellectual state of the individuals of this race. Now, as it is evident that individuals of low intelligence-people generally devoted to manual labor which fortifies the body-do not weigh less than individuals possessing a more developed level of intelligence, it follows that it is not the variations in body weight that are responsible for producing the observed cranial differences.

Given all of the above consideration, we are therefore able to repeat as a conclusion that body weight very likely has a notable influence on the weight of the brain, but that it is certainly not to this influence that we ought to attribute the profound differences in brain weight that we observe between men and women, between individuals of the same sex and same race, and finally between individuals of different races.

## § 4. Influence of age on the weight of the brain

Making use of numbers generated by research conducted by the German anatomist, Rudolf Wagner, Doctor Broca has shown that age exerts a certain influence on the weight of the brain. From 20 to 40 years of age the brain increases in weight, than remains stationary up to age 50, and will constantly decrease thereafter. The maximum difference between the average weight of the heaviest brains-that is to say, those of individuals 40 years of age-and the average weight of the lightest brains-that is to say, those of individuals age 60 and older-was found to be only 84 grams, which is even less than the average brain weight difference I discovered between the shortest and tallest subjects. Although age has an influence on brain weight, this influence is, as one can see, minimal in the presence of differences ranging from 600 to 700 grams observed between the brains of individuals belonging to the same race.

As the brain can only become larger on condition that the bony cavity which contains it is able itself to expand, and given that the brain appears to increase in size up to age 40, it must certainly be admitted that the skull continues to expand as well up to this period in life. Now, some have formerly objected to this contention by noting that many Englishmen have gone over to India, have had their hats made there, and their hat sizes have not changed over several years; but, to this one can respond that the British who visit this colony generally do not remain there for a very long time, and that the number of hatmakers there who supply hats directly to England, instead of those who address themselves to resident English tradesmen in India, must be too small for one to be able on such vague facts to base serious
conclusions. Head size data that I have obtained with the assistance of hatmakers have proven to me, on the contrary, that measurements taken of an individual during a certain time will not necessarily match him later on. In fact, I know a hatmaker who has undeniably verified on himself that from age 23 to 30 his head had gained 2 centimeters in circumference.

## § 5. Influence of race and level of civilization on the weight of the brain

If, as we intend to prove, the unequal development of the brain that we observe between men is mainly related to inequalities of intelligence and sentiments, we must expect to find considerable differences not only between different races, but also between individuals of the same race, according to the degree of their having become civilized. Observation proves that this is the case, and that these differences are distinctly apparent, even when one restricts himself to comparing the averages which efface, as we know, the extreme inequalities. But, it is above all when one groups in series the skull volumes, and looks for how many skulls there are in each race belonging to each of these series, that one sees manifested the differences that mark themselves out to us and which incompletely appear whenever one limits himself to comparing the averages.

The numbers in the following table and the curves whose construction they have served very clearly show these differences and prove that what constitutes the superiority of one race over another is not only a more or less greater superiority in the average capacity of its skull, but certainly also the greater number of voluminous skulls that the higher race contains. In examining the inferior races, one sees that their most voluminous skulls never exceed a certain size. For example, in combining the skulls of Australian aborigines and Pariahs of India, one does not find a single individual whose skull capacity surpasses 1510 cubic centimeters. Additionally, in examining all the discovered skulls of the inferior races (Negroes, Bushmen, Australian aborigines, Pariahs, etc.), I have not encountered a single skull whose capacity measures between 1700 and 1800 cubic centimeters,
whereas with present-day Parisians there are 11 out of 100 which fall within these limits. Although the difference in the average skull capacity between the most elevated races and lowest races barely exceeds 200 cubic centimeters, the difference between the largest skulls of the superior races and the largest skulls of the inferior races comes to 400 cubic centimeters.

## TABLE VIII

Skull Volume in the Human Races*

| Cranial Capacity <br> (cubic centimeters) | Present-day <br> Parisians | $12^{\text {th }}$ Century <br> Parisians | Ancient <br> Egyptians | Negroes | Australian <br> Aborigines |
| :--- | :---: | :---: | :---: | :---: | :---: |
| 1200 to $1300 \ldots$ | 0.0 | 0.0 | 0.0 | 7.4 | 45.0 |
| 1300 to $1400 \ldots$ | 10.4 | 7.5 | 12.1 | 35.2 | 25.0 |
| 1400 to $1500 \ldots$ | 14.3 | 37.3 | 42.5 | 33.4 | 20.0 |
| 1500 to $1600 \ldots$ | 46.7 | 29.8 | 36.4 | 14.7 | 10.0 |
| 1600 to $1700 \ldots$ | 16.9 | 20.9 | 9.0 | 9.3 | 0.0 |
| 1700 to $1800 \ldots$ | 6.5 | 4.5 | 0.0 | 0.0 | 0.0 |
| 1800 to $1900 \ldots$ | $\frac{5.2}{100.0}$ | $\underline{0.0}$ | $\frac{0.0}{100.0}$ | $\underline{100.0}$ | $\underline{100.0}$ |

[^0]These differences of several hundred cubic centimeters between the largest skulls of the superior races and the largest skulls of the inferior races are of capital importance, and I cannot stress this enough. They constitute, in effect, a veritable chasm that nothing is able to fill. If one grants as valid, as I shall soon demonstrate, that the largest brains generally belong to distinguished men, one will easily acknowledge, I believe, that a race of mediocre average intelligence which contains 5 distinguished men out of every 100 will be much superior to a race possessing a
higher average intelligence, but which does not possess a single distinguished man. In reflecting upon the importance of scientific and industrial discoveries, which only superior men are able to produce, no one can contest, I believe, this assertion, and most everyone should readily agree with Doctor Broca that "if there are in each century ten men like Newton, our civilization will advance in proportion to the growth of the sciences." A race that is composed, for example, of individuals who all have a skull volume totaling 1500 cubic centimeters will obviously be quite inferior to a race where 90 out of 100 individuals only possess a skull volume of 1400 cubic centimeters, but where the other 10 out of 100 individuals have a skull capacity of 1700 cubic centimeters. In restricting oneself, as is commonly done, to only comparing the averages between two races, one will make the mistake of declaring, on the contrary, that the first race is much superior to the second; this occurs because the average skull volume of the first race is, in fact, 1500 cubic centimeters, and that of the second only 1430 cubic centimeters.

In Table VIII that I have provided above, which concerns the state of the development of the skull in the human races, one sees that many important races do not appear in it, and I make no pretension that this table is complete. It only forms a sort of canvas whose contours will become clearer over time. Although we have in our hands skull volume measurements for other races, we have not attempted to expand on Table VIII because these measurements seem to us to be insufficiently numerous.


South African Negro skull
The skull volume of African Negroes always measures less than 1700 cubic centimeters.


Plate V. - Curves showing the progressive development of skull volume in the human races and clearly indicating that there are a great number of men who by skull volume are nearer to the apes than to other men.

The left side scale is the scale of skull volume from 1200 to 1900 cubic centimeters. 1 millimeter $=10$ cubic centimeters; 1 centimeter $=100$ cubic centimeters.

One need only count how many millimeters are horizontally contained between the points where the curve intersects the horizontal lines corresponding to the level of the left margin numbers in order to learn how many out of 100 subjects there are having a given cranial capacity. For example, let's say one wishes to know for every 100 present-day Parisians how many possess skulls measuring from 1800 to 1900 cubic centimeters. One sees immediately that between the points where the curve cuts the two horizontals corresponding to the numbers 1800 to 1900 the distance measures 5.2 millimeters. This amount represents the sought after number.

One might reproach us, however, for not having placed in equal light any contrary facts to the theory that we espouse; for this reason we shall not omit remarking that among the measurements that we have not utilized there seems to exist proof that certain races which cannot be placed on the highest levels of the scale of civilization possess nevertheless a more voluminous skull than the superior races. For example, Doctor Broca's measurements show that the average skull capacity of the ancient Gauls is greater than that of contemporary Parisians. The capacity of Finnish skulls, according to measurements effectuated at the Anthropology Laboratory on 41 skulls belonging to the Helsingfors Museum, also present an analogous superiority. Although the measurements that we possess of these two races are not in general sufficiently numerous for us to draw from their study any precise conclusions, I quite willingly acknowledge, however, this cranial superiority whose existence, I believe, can be easily explained. The two races I just mentioned were or are less civilized undoubtedly than present-day Parisians, but they have not played a less considerable role in history, and if it is necessary to establish a parallel between the bold Gallic warriors, for example, who for a long time made Rome tremble ${ }^{2}$, and the present-day Parisian, represented by a clerk at his desk or a factory worker, I hardly know in favor of whom the balance will incline. One must, moreover, consider-and this is an important point-that the development of the brain is not in conformance only with one's level of intelligence. It also corresponds with the development of an individual's personality traits, and we can easily comprehend how a race in whom energy, bravery, the spirit of initiative, and a strong sense of independence are highly developed can possess a more capacious skull than another perhaps more intelligent race, but one which possesses these traits to a significantly lesser degree.

By studying the curves of Plates V and VI, one sees that they provide us with more than just the true sense of the real differences which exist between men. Indeed, these most revealing curves also place in evidence the following extremely important fact which, I believe, has never before been pointed out: it is that the skull volume differences which exist between individuals
belonging to the same race are much greater the higher the race is in the scale of civilization. It will be sufficient to show, for want of other proofs, that it is neither height, nor weight, nor age that account for these gradational differences in skull volume (or, for what amounts to the same thing, the differences in brain weight existing between men).


Plate VI. - Increase in the difference that one observes between the least voluminous and most voluminous skulls of each race, as one goes up the scale of races and as the same race becomes civilized.

The left margin scale represents cubic centimeters. 1 millimeter $=10$ cubic centimeters; 1 centimeter $=100$ cubic centimeters.

In Plate VI the difference between the least voluminous and most voluminous skulls (from left to right) of gorillas, Pariahs of India, Australian aborigines, ancient Egyptians, $12^{\text {th }}$ Century Parisians, present-day Parisians, and Germans are displayed. The numbers written parallelly to each vertical line indicate the largest differences in cranial capacity that one observes in each race. The heights of these vertical lines are proportional to these differences.

After having grouped the skulls of male subjects of different races, taking care to establish a comparison of only sufficiently numerous series in order that the terms could be connected in a gradual way, I was able to construct the following table which clearly gives proof to the proposition I just enumerated above.

## TABLE IX

## Differences between the Least and Most Voluminous Skulls in Each Race

Among the gorillas ${ }^{1}$. . . . . 148 cubic centimeters
Among the Pariahs of India ${ }^{2}$. . . 277
Among the Australian aborigines ${ }^{3}$. 307
Among the Ancient Egyptians . . 353
Among the $12^{\text {th }}$ Century Parisians . 472
Among Present-day Parisians . . 593
Among Present-day Germans ${ }^{4}$. . 715
${ }^{1}$ From Doctor Paul Topinard's research.
${ }^{2}$ From cubages recently effectuated at the Anthropology laboratory on a series of Pariah skulls on display at the Anthropology Exposition in Paris. The individuals designated under this name represent the lowest social stratum of India.
${ }^{3}$ From cubages derived by Doctor Broca on a series, to which I have joined Australian aborigine skulls sent to the Exposition which Doctor Callamand cubed.
${ }^{4}$ From the collection (Die anthropologischen Sammlungen Deutschlands) of Doctor Schaaffhausen, of which I have eliminated the largest skull and the smallest skull, as they were not connected to the series by means of gradual transitions. The criticisms that I have leveled earlier on the numerations of this series extend to the cranial capacity differences that it presents, because the connections one might submit the numbers to will only augment these differences. For example, if, by disregarding the soundness of the reason I stated above, I had not eliminated the largest skull and smallest skull from the series, the difference in skull capacity would have been enormous, amounting to 895 cubic centimeters.

All the other skulls from collections that have served to form this table have been cubed by Doctor Broca and belong to the Museum of Anthropology.

One sees from this table that the differences one observes between the largest and smallest skulls of each race are more than double in the superior races from what they are in the lower races. These differences constantly increase in proportion as the race becomes more civilized, and it is in this manner that they are greater in the Parisians of our day than in their ancestors of 600 years ago.

This very important fact leads us to this equally important conclusion, which is that, far from tending towards equality, men tend on the contrary, given the continuance of present conditions, to differentiate themselves more and more. Differences of 600 and 700 cubic centimeters in cranial capacity that we observe between individuals of the superior races, differences which will be even more considerable if we compare men of the diverse races grouped together, are truly immense and appear even more immense when we reflect that, with the skull volume often attaining 600 cubic centimeters in the gorilla, it follows that there are a large number of men who are nearer to the anthropoid apes by the volume of their brain than they are to other men. This here is a result that a comparative study of the averages will never be able to forecast.

If we are obliged to discuss here questions regarding the classification of living species, there exist interesting deductions to draw from the preceding. Most anatomists today concede that the greatest differences which separate men from the anthropoid apes principally deal with the difference in the volume of their brains; but, because there are between the brains of diverse men differences in volume larger than those which separate the most inferior men from the gorilla, we find ourselves enlocked in the following dilemma: either the distinctions based on brain volume are not sufficient in order to justify the separation that we have established between man and the ape, and then it is necessary to acknowledge their close relationship, or they are sufficient, and in such a case, given that the differences which separate men amongst themselves are larger than the differences which separate them from the gorilla, it will be necessary to admit that certain human races ought to lose the name of men and should only be
viewed as animal species composed of intermediate echelons between the anthropoid ape and the civilized individual who has reached the highest point of development.

Setting aside these insights whose development we shall entertain later on, and returning to our previously-enunciated fact that the cerebral inequalities existing among men tend to increase instead of diminish, I must remark that this anatomical fact is hardly surprising from the psychological point of view, and it is easy to understand that if intellectual equality is possible between inferior individuals, it becomes more and more impossible as one rises in the scale of the races. Let us take, for example, a present-day peasant who is unable to read or write, who has never been away from his fields, who only possesses a few hundred words in his vocabulary ${ }^{3}$ and has parents similar to him. How, I ask, can one say that this peasant is superior to his Stone Age ancestors who were certainly compelled to deploy more intellectual resources than him in order to live? Clearly, there isn't any reason for his brain to have progressed over the centuries. It therefore remains at most stationary, and, as the educated man constantly improves himself and bequeaths to his lineage the gradually acquired improvements, what results is that the difference in brain volume between the peasant and educated man constantly increases. For want of even the previously-described anatomical proofs, we have been able to arrive to the conclusion I've already stated, which is that far from tending towards equality, individuals of the same race tend in the civilized nations to differentiate themselves more and more.

The fact highlighted above-that the skull volume difference found in present-day Parisians is notably larger than what it was 600 years ago-is one of the most convincing examples that one is able to cite in support of the progressive differentiation of individuals belonging to the same race. The measurements from which I have drawn this conclusion have been carried out on two collections owned by the Paris Museum of Anthropology, and are formed, one by skulls originating from a Parisian cemetery built sometime before the $12^{\text {th }}$ Century, the other by skulls from a modern cemetery. The first contains 67 male skulls; the second, 77
of the same sex. The curves of Plates V and VI and the numbers provided in a preceding table show the skull volume gained over 600 years. One sees that the most numerous modern-era Parisian skulls fall between 1500 and 1600 cubic centimeters in capacity, whereas the most numerous $12^{\text {th }}$ Century skulls fall between 1400 and 1500 cubic centimeters. One also sees that with respect to the modern-era Parisian skulls there are 5 out of 100 whose volume measures between 1800 and 1900 cubic centimeters, whereas one does not encounter a single $12^{\text {th }}$ Century skull that attains this number.

## § 6. Relation between brain volume and the level of intelligence

The most contradictory opinions have reigned in science concerning the relation between the weight of the brain and the level of intelligence. Aristotle was persuaded that in the animal kingdom man is the one who has the smallest head relative to body weight; he also asserted that in our species it is the large-headed individuals who are the least intelligent.

Notwithstanding the authority of this illustrious philosopher, such ideas were much too notoriously erroneous in order to be able to subsist for a long time, and since Galen the near totality of observers are in accord in recognizing the relation that exists between the development of the skull and the level of intelligence. Just a few observations, moreover, are sufficient to recognize that as one descends the scale of living beings the capacities of the skull become smaller and smaller. The numbers published by various authors since Cuvier, and which figure in all the anatomy treatises, leave no doubt on this point. From about 1500 cubic centimeters in the European, the average cranial capacity falls to 550 cubic centimeters in the gorilla, 300 in the lion, 150 in the ram, etc.

With regard to the examination of the relation existing between the development of the brain and the level of intelligence, no longer with respect to diverse species, but solely respecting the human race, authors have expressed since the beginning of this
century very different opinions. For example, Gall owned as an absolute principle the following: it is only in a head possessing a considerable dimension that a distinguished genius is able to lodge, and that below a certain skull volume one only encounters idiots. Parchappe, on the other hand, a victim of calculation errors pointed out by Doctor Broca and who had drawn from his numbers conclusions totally contrary to those that they had substantiated, allowed that among the causes which can make the volume of the head change "the weakest is the development of the intelligence." Gratiolet also held an analogous opinion.

The only way to accurately judge the validity of these contradictory assertions is by having recourse to numerous precise measurements. Carried out over a number of years by anthropologists, such measurements have shined a bright light on this question. Because these numerous measurements, in fact, prove that the most intelligent human races possess a voluminous skull, and that the least intelligent races have on the contrary a very slight cranial capacity, it is evident that there exists a certain relation between skull volume and the development of intelligence. The tables concerning the development of skull volume in the different human races that have been provided earlier in this Paper demonstrate the correctness of my preceding point.

In order to render this relation even more evident, I have thought that the best method will consist of researching whether individuals belonging to the same race who present evident intellectual differences will not also present considerable differences in cranial capacity. Parchappe and Doctor Broca have already effectuated these comparisons on the heads of scholars and ignorant persons, and discovered that the cranial circumference of the former surpassed that of the latter; but, executed on a limited number of individuals, these measurements do not escape the same criticisms that one might lodge on all research carried out on series too small for one to be able to be certain of having eliminated the influence of exceptional cases. These criticisms will, in fact, be even more well-founded with respect to individuals possessing absolutely equal head circumferences, as such individuals might-as we shall see in the next chapter-present
differences of more than 200 cubic centimeters in the volume of their skull. I shall show, in the same chapter, how and why these differences are obliterated when, instead of operating on a very small number of individuals, one acts on a very great number, and that there really exists then a direct relation between the circumference of the skull and its volume.

In order to come up with some series composed of numerous measurements, I thought that the best way to accomplish this was to appeal to a category of tradesmen-the hatmakers-who by profession are obliged to measure the circumference of the head of a large number of persons, and to carry out this operation with exactness, because solely on this exactness depends the acceptance of their merchandise. After some searches I ended up finding one hatmaker, intelligent and with many customers, who kept very accurate record books in which he had entered the names of his clients, their profession and their head size. This size was expressed in special units specific to the hatmaker trade, but which by a very simple mathematical operation that I shall discuss in a later chapter can be exactly converted into centimeters of circumference.

The hatmaker's record books having been placed at my disposal, I devoted myself to the long work of methodically counting up, calculating and classifying according to professions the head circumferences of about 1200 individuals. These individuals were divided into the following categories: scientists and scholars, middle-class Parisians, nobles of ancient families, and domestics. With these elements I have constructed the following table which itself has served to establish the curves in Plate VII.


The size of this man's top hat can be converted into centimeters of circumference via a simple mathematical operation.

## TABLE X

Comparison of the Development of the Head in the Diverse Social Categories of the French People

| Head Circumference <br> (in centimeters) | Scientists <br> and Scholars | Middle-class <br> Parisians | Nobles of <br> Ancient Families | Parisian <br> Domestics |
| :---: | :---: | :---: | :---: | :---: |
| 52 to 53 | 0.0 | 0.6 | 0.0 | 1.8 |
| 53 to 54 | 3.0 | 1.9 | 3.7 | 5.4 |
| 54 to 55 | 4.0 | 6.2 | 9.2 | 5.4 |
| 55 to 56 | 6.0 | 14.0 | 12.8 | 33.9 |
| 56 to 57 | 18.0 | 24.5 | 28.5 | 42.8 |
| 57 to 58 | 36.0 | 24.5 | 22.0 | 10.7 |
| 58 to 59 | 18.0 | 14.9 | 12.8 | 0.0 |
| 59 to 60 | 8.0 | 7.6 | 8.3 | 0.0 |
| 60 to 61 | 6.0 | 3.3 | 1.8 | 0.0 |
| 61 to 62 | 2.0 | 1.8 | 0.0 | 0.0 |
| 62 to 63 | $\underline{0.0}$ | $\underline{0.7}$ | $\underline{0.9}$ | $\underline{0.0}$ |
|  |  | 100.0 | 100.0 | 100.0 |
| 100.0 |  |  |  |  |

One will notice that below the curve of the Parisian domestics of Plate VII exists a curve of the cranial circumference of peasants whose components are not found in the preceding table. Not having succeeded in procuring in the countryside documents similar to the ones that I have obtained for Paris, I have nevertheless been able to learn between what customary limits are contained the head circumferences of the peasants of Beauce, a province where I have carried out much research. I have joined together these two limits with a straight line, leaving behind at its extremities any unknown values of whatever very large and very small skulls that are met with in any series.

We shall now examine these curves and see what conclusions we are able to draw from their study.


Plate VII. Head circumferences of diverse categories of individuals,
for the purpose of placing in evidence the relation existing
between the development of the brain and the level of intelligence.
The left margin scale represents centimeters of circumference from 52 to 62.5 centimeters. 1 millimeter of the scale = 1 millimeter of circumference; 1 centimeter (scale) $=1$ centimeter (circumference).

With the spacing of the ordinates being proportional in our curves to the variations expressed in hundredths of the observed phenomena, it suffices, in order to know out of 100 heads of a given category of given circumferences how many there are, to look for how many millimeters vertically separate the points where the curve intersects the horizontal lines corresponding to a given point of the scale. For example, let's say we wish to investigate among the scientists and domestics how many heads measure between 57 and 58 centimeters in circumference. One immediately sees that with the scientists the curve of the head circumference intersects the horizontal lines corresponding to the numbers 57 and 58 of the scale in two points between which are comprised 36 millimeters; for the domestics this amount only totals 11 millimeters. Therefore out of 100 scientists there are 36 whose head circumference measures between 57 and 58 centimeters, whereas among the domestics there are only 11 whose head circumference measures the same.

The differences that the curves in Plate VII place in evidence are quite striking and are much clearer than those that stand out from an investigation of the numbers that allowed for the curves' construction. We observe, in conformance with theory, that the individuals who exercise their brain the most-scientists and scholars-are also those who possess the most voluminous heads, whereas those persons who exercise it the least-domestics and peasants-present, on the contrary, the largest number of small heads.

Contrary to what one might suppose at first, the curve of the heads of subjects belonging by their names to the most ancient noblesse is somewhat lower to the one of the Parisian middle class, but by reflecting a little one understands the reason for this difference. The circumferences of the heads of the middle class were taken in the wealthiest quarter of Paris and consequently belong generally to the most enlightened ranks of the middle class, that is to say, to those who are the best educated and most elevated by their work and who in reality represent today, after the scientists and scholars, the elite of the French population. The remnants of our ancient nobility, altogether possessing the qualities of valor, chivalrous sentiments, etc., transmitted by a long succession of ancestors, disdained the professions and the type of work that could develop their intelligence, and consequently have allowed themselves to be surpassed. Their role in society, moreover, is today quite insubstantial.

One will notice that in an extremely small portion of its route (less than a hundredth part), the curve of the heads of the middle class rises above the one of the scientists, whereas for the near totality of its distance it lies considerably lower. This apparent anomaly is simply the result of the fact that the curve of the middle class heads has been constructed with about 1000 individuals in which there are found many whose profession was not indicated, whereas only 50 subjects were used in the construction of the curve of the scientists. Even if it is conceded that the few quite voluminous heads that we observe in the middle class do not belong to scientists, one can easily understand that the exceptional cases that are met with in a considerable
number of individuals are unlikely to be observed in a very limited number of subjects.

By examining these different curves one will be able to ascertain that among the scientists and scholars, as well as in the middle class and nobility, one sometimes finds heads as small as those common in the lower classes of society. This fact, which the curves of the skull capacity of different races also place in evidence, proves that if the development of the brain is one of the factors which exercises the most influence on intelligence, this factor is not the only one. Elements whose value we might not appreciate, such as the extent of the circumvolutions, the thickness of the cortical layer, the quality of the cerebral cells, etc. are in direct relation with the development of intelligence and might compensate for any deficiency in brain volume. This therefore accounts for the fact, which I have verified many times, of really high intelligence being contained in heads, undoubtedly not very small, but barely above the average in size. I can cite several distinguished scientists whose head circumference does not exceed 56 centimeters. Also, one of the smallest heads that I have come across ( 53 centimeters in circumference) belongs to a deputy of the National Assembly who, without exactly being a distinguished man, adequately performs his duties. Facts such as this latter one are, however, quite exceptional and in the immense majority of cases the development of the brain coincides, as our curves and tables prove, with the development of intelligence. Moreover, without exception, below a certain number-1000 grams in brain weight, according to Doctor Broca, and 46 centimeters in head circumference, according to Gall-one only encounters imbeciles or idiots. My observations have proven to me that out of 1200 normal individuals you will not find one whose head circumference is less than 52 centimeters. A head smaller than 52 centimeters in circumference will therefore be quite exceptional, and the probabilities are extremely high that its owner will find himself located on some part of the scale between the total idiot and the intelligence reduced to its lowest form.

In this study of the relations existing between the development of the brain and the level of intelligence, I can also invoke this
important fact, which is that the brains of illustrious men that we have been able to weigh after their death possess an elevated weight. These measurements are unfortunately too few in number in order for us to draw from them sufficiently well-founded conclusions. However, it is worth mentioning that the brain of Cuvier weighed 1831 grams and the ones of Byron and Cromwell more than 2000 grams.

The preceding study, in order for it to be complete, requires that I now enter upon the question of understanding what superior intelligence really consists of, but this problem is so complex that it is difficult for me to even touch upon it lightly here. In order for it to be sufficiently treated, it will necessitate developments that will entail my going completely outside the scope of this Paper. Besides, I have already discussed this question at length in my book, l'Homme et les societies, leurs origins et leur histoire (1879), specifically in the chapter titled, Développement physique et intellectual de l'homme. I only wish to remark that it is often very difficult for a not very keen intellect to identify what truly superior intelligence is composed of and to say, for example, what it is between two individuals that makes one superior to the other. It will be neither the more or less greater education received nor the degree of success obtained in life which will be able to serve us as a guide. As far as success is concerned, it is, in fact, quite evident that a brilliant intelligence, but one not accompanied by certain qualities of character such as perseverance, boldness, etc., for example, will be much less successful than one possessing a significantly less elevated intelligence, but who has at his service a highly tenacious personality and knows how to concentrate all his faculties on a single matter. The most famous specialists often only possess a limited intelligence, but one which is accompanied by a very large amount of perseverance. With regard to education, it is no less evident that it is not the large sum of knowledge that an individual has been able to acquire which serves us as a guide in order to appreciate the state of his intelligence; rather, what matters most is the individual's creative faculties, not his memory. Genius inventors frequently have at their service only a very small sum of knowledge compared to that
possessed by highly obscure individuals who deserve to remain obscure.

If it is absolutely necessary to spell out in a few words a formula to measure intelligence, I would say that it can be appraised by the degree to which one possesses the aptitude to associate-mind you, I did not say to accumulate-the largest sum of ideas and to perceive, in the most clearly and rapidly possible way, their analogies and differences. An inferior intelligence will hardly be able to associate more than two ideas at a time, and will not see their differences or apparent analogies. The Eskimo knows that ice melts in the mouth; he then sees a piece of glass, a substance that resembles ice, and immediately concludes that glass must also melt in the mouth. Such associations and analogous ideas form the foundation of all the beliefs of inferior beings. For example, among certain peoples it is believed that by eating the flesh of the tiger one acquires the courage of this animal, and on the other hand one exposes himself to becoming faint-hearted like the deer when one feeds on the flesh of this latter animal.

As one elevates himself higher and higher up the scale of civilization, one sees an expansion of the aptitude to associate a larger and larger number of ideas, and to perceive through the appearances their actual resemblances and differences. The whale much more resembles a fish than a horse or a bat, and yet the scientist knows that between the first two the resemblance is only apparent and, thanks to his aptitude to associate by thought a large number of qualities, he sees that the whale is much more a relative of the horse and bat than of the fish.

When the aptitude to associate ideas and understand their connections is highly developed, it leads to the greatest discoveries. It is because this aptitude was possessed to a high degree that Ocken discovered the analogy respecting the skull and vertebrates; Goethe, that relating to the flower and leaf; Davy, the one with respect to potash and metallic oxides. When Newton identified the fall of a heavy body onto the ground with the attraction which is exercised on celestial bodies, and Franklin considered the spark
and lightning as manifestations of the same phenomena, they put into play this faculty. It is what shows to the general on the battlefield, as well as to the scientist in his laboratory, what are among the possible associations of elements that he has in his hands the best combinations which will provide to one victory, to the other the discovery of a new fact. This aptitude is almost entirely absent in women, and it is its absence which largely results in their lack of logic, their incapacity to reason, and their habit of only being guided by the impulse of the moment.

Given the quite evident relations that we have proven exist between the development of the head and the level of intelligence, do they permit us to affirm that the brain expands in size in the adult who exercises his intelligence? To answer this question we shall first recall that it is a well-known physiological law that every exercised organ increases in volume, and on the other hand atrophies when it ceases to function. It is therefore exceedingly probable that the brain is no exception. Although very probable, this fact has not yet been experimentally demonstrated. I do not believe, in fact, that one may invoke in favor of this progressive development the fact that individuals engaged in the liberal professions possess a more voluminous skull than those belonging to the socially inferior categories. Certainly, those differences in skull volume have been able to increase by the nature of the particular work engaged in by the aforementioned individuals but, unless one contests the influence of heredity, there is no doubt that they have been at least in very large part furnished beforehand at birth. The data that we have provided earlier in this Paper show how at the moment of birth the variations in the circumference of the head are already considerable. It is only by comparing over a 10-year time period the heads of a series of scientists and illiterates that one might prove experimentally the influence of intellectual work on the development of the brain. Such a comparison, however, has not yet been made.

## FOOTNOTES

1. I have not listed, as one can see in the table comparing the male and female skulls of diverse races, the German skulls of which we nevertheless
possess a quite complete catalog. I have pointed out earlier in this Paper that the numbers published in this catalog are much too deficient and that we cannot make use of them. Persons who desire to repeat the calculations that I have effectuated on the skulls of Doctor Schaaffhausen's collection will find that the difference between the average of the male and female skulls is only 153 cubic centimeters, but it is easy to show by very simple reasoning that this amount is clearly too small. I have already shown that, probably because of measurement errors, the capacities indicated for all the skulls in this collection significantly understate reality. Let us assume the most favorable hypothesis, which is that the measurement error has been for all the skulls the same fraction of their volume. It is evident that this error, which affects equally the volume of these skulls, affects very unequally their differences. Let us compare, for example, two skulls possessing respectively 1300 and 1500 cubic centimeters in capacity and, consequently, presenting a difference of 200 cubic centimeters. Let us now admit that one has committed on each an error of $1 / 10^{\text {th }}$ of their volume; after carrying out the correction, the capacity for the first skull will become $1300+130=1430$ and for the second $1500+150=1650$. Their difference in volume is now 220 cubic centimeters instead of 200 . One sees that, just by the sole fact of having augmented by $1 / 10^{\text {th }}$ the two skulls' volume, their difference in capacity has increased. It is therefore evident that the difference between the German male and female skulls derived from Doctor Schaaffhausen's measurements is way too small. I willingly admit, however, the following fact which lends confirmation to the explanation I give of one key cause of the difference existing between male and female skull volumes: in countries such as America or Germany, where women receive a clearly superior education to that which they are provided in Latin nations, the difference existing between the skull capacity of men and women must tend to be reduced. The numbers that I have cited above representing the difference that exists between the German male and female skulls will tend, even after their being submitted to a considerable correction, to confirm this hypothesis.
2. It was said of the Gauls by the historian Salluste that when our forefathers fought the Romans, they didn't do it for the glory, but for the principle. Certainly, the Romans were much more civilized than the Gauls, but memories of the sustained struggles against them were so terrible (such as that of the shocking defeat in 104 B.C. of Quintus Cæpion and Cneius Mallius to our courageous ancestors) that within the Roman law which accorded exemption from military service to priests and old men, it was stipulated that this exemption would cease at the time of war against this people; and it in effect required all the immense power of the Roman civilization, and all the genius of Caesar in order to triumph over these barbarous hordes who only had for themselves their intrepidity and their valor.
3. The English peasant, according to Müller, only possesses 300 words in his vocabulary. In order to understand all the words of the simplest book, such as the Bible, it is necessary to know 5,000; for reading Shakespeare, or a poet
like Milton, one needs to know eight thousand. Needless to say, the learned man is obliged to know a considerably larger quantity.


An English peasant, with his extremely limited vocabulary of 300 words, would not be able to read Shakespeare's plays, let alone understand even the simplest books.

## V. Inequalities in Volume of the Corresponding Regions of the Skull

The skull is a very asymmetrical organ. - This asymmetry does not correspond to that of the other parts of the body. - It does not appear more frequently on one side than the other. - The skull's lack of symmetry is not of the same sense for each of its parts.

During my studies that $I$ have pursued for a long time concerning the variations of the brain's size and shape that one observes among individuals belonging to the same race, I have had occasion to investigate whether the different parts of the cerebral hemispheres on the right side and on the left side habitually possess the same size.

Not being able to easily obtain a determination of the weight of the brain and its associated parts, I was obliged to effectuate my research upon the skull itself. My measurements have been taken on nearly 300 skulls belonging to different series in the collection of the Museum of Anthropology, which were graciously placed at my disposal by Doctor Broca.

For a long time anatomists have wondered whether the two cerebral hemispheres are quite alike. The most widely-held opinion has been that of Bichat, who considered that a lack of symmetry of these organs must be accompanied by a lack of sound judgment. The autopsy of this famous anatomist, whose own skull proved to be most irregular, demonstrates how little this opinion is well-founded.

With man, the majority of the organs are generally more developed on the right side than on the left side; but, considering that the left portion of the brain presides over the functions of the right part of the body, one might deduce that it is the left hemisphere of the brain which must be the most developed. A professor of Bordeaux, Doctor H. Fleury, recently affirmed a similar opinion, based on his contention that blood circulation is more active in the brain's left hemisphere than in the right hemisphere, because of the dissymmetrical divisions of the aortic arch.

Notwithstanding their rational appearance, these theories have not been confirmed by observation. With respect to the 287 skulls that I have measured, in choosing as a reference mark the vertical plane passing through the external occipital protuberance and the extension of the middle ridge of the nose bone, I have procured the following results:
Skulls where the right side predominates over the left side ..... 125
Skulls where the left side predominates over the right side ..... 111
Skulls whose various bones are unequal, but whose inequalities offset each other, so that the right half is nearly the same as the left half ..... 51

There is, as the above results indicate, an advantage on the right side of the skull, but it is slight; and, in reality, the skull is sometimes more developed on the right-then again, sometimes on the left, without it being possible to assign any solid reasons for this inequality of development.

From the very first I have believed that the inequality of development of the homologous parts of the skull would be found more frequently on the left than on the right side among intelligent subjects; and observations made with the conformator (that is, the hatmakers' head-measuring instrument) on more than 200 living heads led me to instantly be persuaded in the accuracy of this hypothesis. But, I have discovered since then that by reason of the difficulty of successfully positioning the large axis of this instrument, one cannot rely upon its indications. I have in the meantime preserved in an album, which is available to anyone who might find it interesting, 200 drawings taken upon the living with aid of the conformator.

Plate VIII expresses in a schematic fashion the inequalities that one observes the most frequently. The combinations that might present themselves are much more considerable than the ones that I have represented, given that there are six elements that can associate themselves on the two sides of the skull.


Plate VIII. - Diagrams portraying the most frequently observed inequalities of development of the diverse bones of the skull.

The most frequently observed inequalities of development of the diverse bones of the skull are (clockwise, from upper left) as follows:

1) Frontal predominant on the right side with parietal predominant on the left;
2) Frontal predominant on the left side with parietal predominant on the right;
3) Frontal equal on both the right and left sides, but with one of the parietals predominant;
4) Frontal and parietal predominant on the same side with the occipital predominant on the opposite side.

Now, what regions of the skull show the unequal development that I have pointed out? A priori, it will seem that it must manifest itself in the same sense for all the bones belonging to the same side; but observation will once again contradict this hypothesis. When, for example, the frontal bone predominates on the right, one very often sees the parietal predominating on the left and vice versa. Whenever it happens that the parietal and the frontal predominate on the same side, one can be fairly certain that the occipital will predominate on the other.

Whenever the two halves of the skull seem equal, as in the third case mentioned in the preceding table, it is because the inequality and the irregularity of certain bones on one side have been compensated for by inequalities on the opposite side; the predominance of the parietal on the right, for example, will be offset by the predominance of the occipital on the left half, and the two halves of the skull will appear symmetrical; but, in none of the nearly 300 skulls that I have measured have I ever found all the corresponding parts of the right and left side equally developed.

The preceding observations demonstrate that the skull, and probably the brain as well whose shape it has reproduced from its own, presents a lack of constant symmetry that is not of the same sense for each of its parts. For now, though, I shall confine myself to establishing this important anatomical fact without trying to draw at this time any physiological inferences.


Typical skull possessing unequal corresponding regions

## VI. Mathematical Relations between the Diameters, Circumferences, and Volumes of the Head and Skull

Importance of conducting research on the living in order to understand the relations that exist between different parts of the skull. - § 1. Relations existing between the antero-posterior and transverse diameters of the head and its circumference. - Mathematical formula linking these two dimensions. - Table showing the errors committed in making use of this formula. - § 2. Relations existing between the circumference and volume of the skull. - Mathematical reason for the impossibility of this determination in isolated cases, but the possibility of its determination for the series. - Table providing the volume of the skull when one knows its circumference. - § 3. Relations existing between the circumference of the head, the circumference and volume of the skull, and the weight of the brain. - Method employed for determining these relations. - § 4. Relations existing between the diameters of the skull and its volume. Discussion of these relations. - Reason for the impossibility of finding a formula permitting one to express the volume of the skull in terms of its diameters.

Determining the volume of the skull in terms of its measurable elements on the living constitutes a problem whose solution is most important from the point of view of anthropology. It is evident, in fact, that as long as it will be possible to determine the volume of the head only on skulls instead of ascertaining it from the living, the number of measurements that one will be able to collect will always be quite limited. Because skulls that museums possess nearly always originate from unknown individuals, their measurements clearly have much less interest than if they originated from living subjects whose intelligence and diverse aptitudes are well understood. Furthermore, it is also evident that it is only when the skull's volume will be able to be easily determined on the living that explorers will be in a position to report useful information on the cranial capacities of the races visited by them.

We shall now proceed to show that by working, not on isolated cases, but on a series, that it is possible to determine with sufficient exactness the volume of the skull by means of quite easy-to-execute measurements on the living. In two hours a moderately competent explorer will be able, by means that we shall soon describe, to calculate the skull volume of a hundred individuals of one tribe. Divided by groups, each differing by 100
cubic centimeters in capacity from the preceding or following group, measurements effectuated in such a manner on a large scale will most assuredly provide more useful information than the cubage of a small number of skulls that one procures with difficulty.

We shall now successively examine the relations existing between the diverse diameters and circumferences of the head and skull, as well as those pertaining to the skull's volume.

## § 1. Relations existing between the antero-posterior and transverse diameters of the head and its circumference

One might believe at first that there is little interest in knowing the relations that exist between the diameters of the head and its circumference, because it is always possible to directly measure this latter; but, even if one admits-what might be strongly contested-that the measure of the head's circumference is subject to fewer errors than that of its diameters whenever one possesses a suitable instrument, I have in regard to my research an interest to the highest degree in understanding the relations that exist between the two horizontal diameters of the head and its circumference, because it is from the understanding of the former that I was able to deduce in succession first the circumference and then the volume of the skull of about 1,200 living subjects.

I have stated in the preceding chapter that the circumferences of 1,200 Parisian subjects, percentage values of which I have provided in Table $X$, were derived from measurements carried out by a hatmaker. Everyone knows the manner in which these tradesmen operate in order to take an extremely precise measurement of the head that they must dress up. It consists of measuring with a kind of retractable ruler the antero-posterior and transverse maximum diameters of the hat that the buyer is wearing. A very simple empirical method immediately provides in particular units called points the sought-after circumference and enables one to find the proper hat. An instrument called the conformator is then sometimes made use of in order to adapt to the irregularities of the head the hat whose circumference has
already been expressly provided by the aforementioned retractable ruler and whose indications are, as experience proves, sufficiently exact. I have not, incidentally, been able to procure information on the origin of this latter instrument, but it must be very old, as evidenced by the graduated scale in inches which appears on one of its faces.

In researching what grounds, evidently empirical, the retractable ruler's inventor relied upon in order to deduce the circumferences of the antero-posterior and transverse diameters, I have discovered that the measurements that the instrument provides are bound to a relation that one can express by the following equation:

$$
x=\frac{(\mathbf{A}+a)}{2} \times 3.19
$$

$\boldsymbol{x}$ represents the sought-after circumference of the head; $\mathbf{A}$ is the antero-posterior diameter and $\boldsymbol{a}$ is the transverse diameter.

In verifying experimentally the extent of the errors committed with this instrument, and which very rarely exceed $1 \%$ of the sought-after circumference, I have discovered that one will obtain a greater precision by replacing in the preceding formula the number 3.19 with 3.22 . The formula then becomes:

$$
x=(\mathbf{A}+a) \times 3.22
$$

The following table (Table XI), formed with a series of 25 random-selected Parisians skulls from the West Cemetery, but chosen in a way such that subjects possessing very different cephalic indexes appear (that is to say, subjects who are very dolichocephalic and very brachycephalic), gives the comparison between the results obtained on the skulls by direct measurement or through calculation by means of the preceding formula.

## TABLE XI

Calculation of the Circumference of the Head in Terms of its Antero-posterior and Transverse Diameters

| Sum of the two |  |  |  |
| :--- | :---: | :---: | :---: |
| diameters, i.e., | Cephalic Index <br> (ratio of the | Circumference <br> observed | Circumference <br> calculated | | Difference in |
| :--- |
| millimeters |
| between the |



One sees in this table that for 13 skulls the average error is less than 4 millimeters, 11 of which the error did not exceed 3 millimeters. By in large, errors made on the plus side are offset by errors made on the minus side, such that the entirety of the two results can be considered as identical. The calculated average circumference only differs, in fact, from the measured average circumference by 1 millimeter.

If instead of finding the average of all the calculated circumferences one applies the formula to the averages of the diameters, the coincidence will be even more complete because the deduced number totals 511.98 millimeters versus 512 millimeters for the calculated average-a difference of only .02 millimeters. By operating on the data in this manner one can replace 25 multiplication steps with a single one.

I have tested the formula on the averages of cranial diameters of a large number of races and have verified its accuracy; this accuracy, though, is less for female skulls notably than in the preceding case, which one can easily understand when one recalls that all the human heads presented in Table XI were not of the same type and also that the formula has been established for a given race. I shall not reproduce here the table of these measurements, the reader being in a position to easily reconstitute it.

The preceding formula is not as empirical as it may seem which, besides, from the standpoint of the result will be unimportant. It evidently amounts to this: that, with the circumference of the head being an ellipse whose two axes can only vary one in regard to the other within certain limits, the contour of this ellipse is equal to the average of its two axes multiplied by a number a little larger than the ratio of the circumference to the diameter (3.22 instead of 3.14) and whose value through observation one is able to determine.

If, instead of being able to be considered as an ellipse whose two axes little differ, the horizontal section of the head possesses the shape of an elongated ellipse, the preceding formula will obviously not be applicable and will need to be replaced by the
formula of the contour of an ellipse, but this latter formula which belongs to integral calculus is extremely complicated.

## § 2. Relations existing between the circumference and volume of the skull

In examining the tables where the volumes of skulls and their circumferences appear together, it appears at first that there absolutely does not exist any correspondence between these two values. For example, I have found in the series of skulls belonging to the Anthropology Museum subjects possessing the same cranial circumference yet presenting differences in skull volume of nearly 250 cubic centimeters. One can easily understand such differences by reflecting that, besides the circumference, many factors such as, for example, the skull's vertical height, the narrowness or width of its upper part, etc., can modify the volume limited by the circumference. As one knows, solids possessing exactly the same circumference at their base and the same height, such as a hemisphere, a cylinder, a cone, etc., have very different volumes.

Theory and observation therefore seem well in accord to show that there is not any relation to establish between the circumference of the head and its volume. But, this assertion, in appearance so well-founded, and which applies in reality for individual cases, is incorrect when one operates on a series. Whenever one groups a certain number of individuals of a given cranial capacity, and then compares them to individuals with a higher cerebral capacity, one always sees that the average circumference of the former is lower than the average circumference of the latter. By continuing this same operation on a series of individuals of increasing average cranial capacity, one finds that the average of the circumferences of the skulls of each group regularly increases with their volume.

Undoubtedly, nothing seems more contradictory at first sight than the two preceding propositions, and it appears difficult to conceive how the single act of assembling individuals together in groups is able to place in evidence relations which do not exist when one examines isolated subjects.

Even though we do not have any ready explanation for this anomaly, it turns out that, in fact, nothing is more explicable than this apparent contradiction. Firstly, the number of shapes that heads of individuals of a given race can present is clearly not infinite. Being necessarily finite and very naturally very much smaller than the number of individuals of this race, what results is that the same combinations frequently repeat themselves and, when one observes a given number of individuals, one inevitably comes across many times heads possessing quite similar shapes. To continue, let us now suppose that in a particular race the heads assume four different shapes-a spherical dome, a cone, a truncated cone, and a cylinder-and that these four solids possess the same volume. Geometry demonstrates that they will be able to have at their base very different circumferences and that an increase of 1 centimeter in these circumferences will produce considerably unequal increases in the volume. By examining separately the individuals whose head assumes the four shapes I have indicated above, one will be justified to say that there does not exist any relation between the circumference of the head and its volume. Instead of the previous approach, let us now carry out our examination on a group of 100 in which the four shapes will be equally represented and where all these skulls will be increased by 1 centimeter at their base. This increase of one centimeter in the base's circumference will obviously produce an increase in volume identical for all the skulls of similar shape and capacities, but a quite different increase in volume for skulls that have a different shape. Individuals with the same cranial circumference, but having different head shapes, will therefore always possess, whenever one considers them in isolation, very unequal cranial capacities. If we now combine all these measurements and find the average of the volumes and circumferences, we easily see that the increase of these two values progresses in a parallel manner, that is to say, the increase of the number pertaining to the skull's average volume will invariably be accompanied by an increase in the number of its average circumference.

In order to facilitate the preceding demonstration, we have supposed that the shapes that the head might assume are quite limited, which is not at all the case; but, it is evident that the
number of shapes which may present themselves will not alter the value of our demonstration.

One now readily understands why, by operating on sufficiently numerous groups in order that all the cranial shapes find themselves equally represented, the increase in the skull's average volume is accompanied by an increase of its circumference. As the head represents an irregularly-shaped solid, the value of the relation existing between these two dimensions will not be genuinely determined but by experiment.

In order to approximately determine the interconnection that exists between the circumference of the skull and its volume, I have grouped by order of increasing capacity 87 male skulls from the West Cemetery, and found the average of the circumference and volume of each group; this has led to my obtaining the following table.

## TABLE XII

Relations between the Circumference of the Skull and its Volume*

| Cranial Capacity | Average Circumference |  |
| :---: | :---: | :---: |
| (cubic cm.) | (millimeters) | Increase in the Circumference <br> for each 100 cc. <br> Increase in Volume |
| (centimeters) |  |  |

From 12 to 1300
From 13 to 1400
From 14 to 1500
From 15 to 1600
From 16 to 1700
From 17 to 1800
From 18 to 1900

| 489 | $?$ |
| :---: | :---: |
| 503 | 1.4 |
| 512 | 0.9 |
| 521 | 0.9 |
| 531 | 1.0 |
| 544 | 1.3 |
| $?$ | $?$ |

* Because none of the male skulls in the series forming this table were less than 1300 cubic centimeters, I have formed the average circumference of the lowest skull volume group ( 12 to 1300 cc .) with female skulls originating from the same cemetery. All the other numbers appearing in the table have been solely established with male skulls.

One sees from the preceding table that when one operates on a series of skulls, an increase of around 1 centimeter in the circumference produces a nearly 100 cubic centimeter increase in the volume.

Because the head approaches by its shape certain solids of revolution (i.e., solids rotated about an axis), it must naturally be a party to the properties of these solids, and on heads of unequal diameter an increase of one centimeter in the circumference will clearly not produce the same increase in volume. For example, the formula $\mathrm{K}=\sqrt[3]{\frac{\sqrt{\times 3}}{4=}}$ shows that for a sphere with a volume of 1438 cubic centimeters an increase of 1 centimeter in the circumference will only produce an increase of 86 cubic centimeters; but, on a sphere 1752 cubic centimeters in size the increase will be 115 cubic centimeters. These two numbers, as one can see, do not appreciably deviate from 100 cubic centimeters.

It is important to note that the preceding calculations are only applicable to the race for which they have been established, and are without value for any other race. Each race requires a table similar to the preceding, constructed upon the same principles. One can conceive, in fact, that the height of the skull, as well as other factors which are too many to enumerate, not being similar in the diverse races, will be able to make the volume, limited by the circumference, vary considerably. This is what explains why certain races can have an average cranial circumference greater than that of other races, and yet possess a smaller average cranial capacity. It is in this manner, for example, that the Eskimos, having an average circumference and height of the skull larger than the ones of the Parisians, nevertheless possess a much less voluminous skull than the latter. The sum of the antero-posterior and transverse diameters likewise favors the Eskimos. It is probable that in large part it is in the way in which these diameters are distributed that the origin of the Eskimos' smaller cranial capacity resides; the Eskimos, as we know, are extremely dolichocephalic, and Parisians, on the contrary, are nearly brachycephalic-that is to say, the two horizontal diameters of the head are considerably less unequal in the latter than in the
former. Now, given two solids of revolution possessing axes whose sum is equal, but whose relative lengths are unequal, we can demonstrate by means of geometry that it is the solid whose axes are the less unequal between the two, that is to say, the solid which most approaches the sphere in shape, whose capacity is the largest.

## § 3. Relations existing between the circumference of the head, the circumference and volume of the skull, and the weight of the brain

The solution to the previously-enunciated difficult problem has been found quite readily by our graphical method. After having obtained by groupings in series, with the previously-given numbers, curves of the circumference of the head, the circumference of the skull and its volume, and then the weight of the brain, by superimposing these curves we have sought the relations existing between them. For lack of a sufficient number of skulls, these relations have not been able to be determined for the outermost regions of the curves. The following values express the results obtained.

## TABLE XIII

Relations existing between the Circumference of the Head, that of the Skull, the Volume of the Skull, and the Weight of the Brain

| Average <br> Circumference <br> of the Head | Probable Average <br> Circumference <br> of the Skull | Probable Average <br> Volume of the <br> Skull | Probable Average <br> Weight of the <br> Brain |
| :---: | :---: | :---: | :---: |
| 550 mm. | 503 | 1350 |  |
| 560 mm. | 512 | 1450 | 1150 |
| 570 mm. | 521 | 1550 | 1250 |
| 580 mm. | 531 | 1650 | 1350 |
| 590 mm. | $?$ | $?$ | 1450 |
|  |  |  | $?$ |

Now, these numbers are only applicable, of course, for the previously-explained reasons to a series, and not at all to isolated individuals-that is to say, that if six individuals have a head
circumference which measures 58 centimeters, the average weight of all their brains will be 1450 grams, without our being able to say that it will be the weight of any one of them.


Plate IX. Variations of the weight of the brain, the volume and circumference of the skulls in the Parisian population, and the relations of these values.

The three curves, as labeled from top to bottom in the central part of Plate IX, are those for brain weight, skull volume, and skull circumference.

The left side scale is for brain weight, from 900 to 1700 grams. The two scales on the right side represent 1) skull volume, from 1300 to 1900 cubic centimeters, and 2) skull circumference, from 49 to 56 centimeters.

In order to extend the preceding table and at the same time to give more precision to the numbers that it contains, it will be necessary to have at one's disposal a larger number of measurements than the ones that we presently possess. The method having already been laid out, it will be easy to apply it to the new measurements.

Such an extension of, and elaboration on, Table XIII will be most useful to anthropologists because these numbers permit us, with the knowledge of very easily measured elements, to determine all the ones on the living that escape our measurements.

For reasons similar to the ones I have given with respect to the relations existing between the circumference of the skull and its volume, the indications of the preceding table are applicable only to the races for which it has been constructed. For different races similar work will need to be carried out.

## § 4. Relations between the diameters of the skull and its volume

Various anthropologists and mathematicians have tried to find a relation between the skull's diameter and its volume, and at the beginning of my craniological studies I myself devoted considerable time to this research. It was only after having become well-convinced that the determination of the volume in terms of its diameters is, as I shall show shortly, absolutely impossible that I employed the previously-explained indirect method in order to infer the volume of the head and its circumference.

Before demonstrating the impossibility of finding a relation between the diameters of the head and its volume, I must say a few words about some research that has been carried out on this point. Doctor Broca, who has also studied this question, arrived, in researching the correspondences existing between the volume of the ellipsoid representing the head and that of the parallelepiped, then amending these correspondences from the experiment's data, to these conclusions: the volume of the skull is equal to half the product of its three diameters divided by 1.12. The average error does not exceed $4 \%$ of the actual capacity in the majority of the
cases, but might go up to $7 \%$, being about 100 cubic centimeters off in certain instances. As the author of this formula has noted, it is only an approximation, and one should take recourse in this formula only when taking the skull's cubage is impossible, given that performing the calculations that the cubic index method entails takes just as much time as finding the cubage itself.

Having experimentally verified that, within the confines where its axes vary, the cranial ellipsoid can be considered as nearly $12 \%$ lower volume-wise to a hemispheric dome having for its diameter an average between the antero-posterior and transverse axes of the ellipsoid, and having for its height the vertical height of the skull, I have been led to express the volume of this organ in terms of its horizontal diameters A and $a$, and of its height $h$, by the following formula:


The letter $n$ represents in this formula a number that is a fraction of the rest of the equation, and whose value has been determined by comparing the volumes measured on the skull with the volumes derived from calculation. I have executed this comparison on a large number of Parisian skulls, and discovered that the value of $n$ oscillates around 12 hundredths. The error habitually committed is generally less than that committed with Doctor Broca's formula, but the result obtained is likewise only approximate.

Whenever I think that it might be possible to determine the volume of the head in terms of its three diameters, I have dreamt of finding a formula such that one can employ it to construct a table which, knowing the three diameters, will permit one to derive without any complicated operation the volume. Theoretical considerations have led me to the following formula:

$$
\mathrm{V}=4\left(\frac{\mathrm{M}-1}{2}\right)^{5}+n
$$

$M$ is the average of the three axes of the head, the quantity 1 represents pretty near what must be subtracted because of the thickness of the bones of the skull; $n$ represents a value equal to $1 / 20^{\text {th }}$ of the result obtained. Applied to contemporary Parisian skulls, whose average diameters are 183, 145 and 132 millimeters, the formula produces for an answer 1545 cubic centimeters instead of 1558. It is unfortunately far from always obtaining a similar precision.

One can easily understand how with this formula, if it had been exact, it would have been possible to construct beforehand tables providing the head's volume by the mere knowledge of the three diameters. In fact, calculations show that with human skulls the value of the factor $(\mathrm{M}-1) / 2$, that is to say, the average interior radius of the skull, lies between 68 millimeters and 84 millimeters. With $(\mathrm{M}-1) / 2=68 \mathrm{~mm}$ and $(\mathrm{M}-1) / 2=84 \mathrm{~mm}$ one obtains volumes corresponding to the maximum differences that one usually observes between the largest and smallest skulls of the elevated races. By solving the equation for all the intermediate numbers falling between the preceding ones, one will possess the elements of a table that gives the corresponding volumes to all the possible human heads.

I have only provided the preceding formulas in order to spare persons who might tackle the subject similar research. I have entirely renounced these formulas' employment as well as research of the same kind, inasmuch as the most thorough study of a large number of measurements taken on skulls has convincingly demonstrated to me that individuals whose cranial diameters are equal often possess very different cranial capacities. It is therefore evident that no formula is able to connect these two cranial values. In the series of Parisian skulls from the West Cemetery that I have most especially studied, there are several skulls which present a significantly larger capacity to that of other skulls, even though their diameters are less. If, as I have done, one arranges these skulls according to increasing capacities, their diverse diameters and the sum of these diameters do not at all follow a regularly increasing course.

The geometric reason is plain for this lack of parallelism between the increase in the volume of the skull and that of its diameter. Whenever the shape of a solid is unknown, it is evident that information about its base and height cannot at all provide any idea about its volume. For example, a cone and a cylinder, although having the same base and height, will nevertheless present a considerable difference in capacity, because the volume of the former will only be one-third that of the latter. Hence, one can easily understand how two heads possessing exactly the same diameters, that is to say, the same base and height, might have very different volumes, according to how closely they approximate the shape of the two previously-cited solids-in other words, according to how narrow or broad their heads are at their top.

The determination of the volume of the head in terms of its three diameters is therefore, when one deals with isolated cases, entirely impossible whenever one can only insert into the formula the values of these three variables. It is probable that by introducing other elements intended to help one determine the shape of the head, such as the length of the antero-posterior and transverse curves, the cephalic index, etc., one might be able to arrive at a fairly certain approximation, but then the formula will be an extremely complicated one and, besides, it will require a lot of research in order to discover and develop it.

The preceding demonstration, keep in mind, only applies to isolated cases. By operating on a series one will be able to derive the volume from the diameters, just as I have derived it from the circumference. I have already explained the reason for this apparent anomaly. A series of calculations that will not be useful to reproduce here have in the meantime proven to me that even by operating on a sufficiently numerous series, the increase of the diameters is less regularly parallel to the increase of the volume than it is to the increase of the circumference.

I shall now end the first part of this Paper, which has been solely devoted to the study of the laws of the variations of brain volume. In a second part I shall take up the difficult study of the
variations in shape of this organ and their relation to sentiments and intelligence.


Cylinder


Cone
Even if two skulls possess the same height and base, as is the case in the cylinder and cone pictured above, their volumes can differ significantly.

## Conclusions

1. The variations of the volume of the skull in the human species are greater and much more apparent than those differences that are visible when one restricts himself to a comparison of the averages. Within the same race these variations are very considerable. For example, by weighing 100 Parisian skulls belonging to the male sex it will be seen that their weight will vary between 1,000 and 1,700 grams. The capacity of an equal number of skulls of the same sex will show that the volume of these skulls ranges between 1,300 and 1,900 cubic centimeters. These extreme amounts are linked to each other in a progressive manner. In mixing together all the races and both sexes, one will recognize that the capacity of the human skull may normally vary nearly $100 \%$, that is, from a simple, small size to one almost twice as large. Many factors, of which the principal one is the level of intelligence, determine these variations or their correlates.
2. The average skull capacity of the superior races considerably surpasses that of the inferior races, but what really constitutes the superiority of one race over another is that the superior race contains many more voluminous skulls than the inferior race. For every 100 modern Parisian skulls examined, there are generally 11 subjects whose skull capacity falls between 1,700 and 1,900 cubic centimeters, whereas in the same number of Negroes one does not find any whose skull possesses the previously-indicated capacities. In the very inferior races, the most voluminous skulls hardly ever exceed 1,500 cubic centimeters. Comparing the largest skulls belonging to the superior races to the largest skulls of the inferior races, the skull capacity difference amounts to the enormous number of 400 cubic centimeters. By contrast, the difference between the average capacity of skulls belonging to these respective races is only a little over 200 cubic centimeters.
3. The aforementioned considerable differences of the brain weight or skull volume between individuals of the same race vary substantially from one race to another. These differences become
greater and greater as the race rises up the ladder of civilization, constantly increasing in the same race in proportion as it becomes civilized. By grouping the volumes of the skulls of each race in a progressive series, taking care to only establish comparisons on sufficiently numerous series in order that the extreme terms are connected in a gradual fashion, one will discover that the difference between the volumes of the largest and smallest adult male skulls is: in the gorilla, 148 cubic centimeters; in Australian aborigines, 307 cubic centimeters; in the ancient Egyptians, 353 cubic centimeters; in $12^{\text {th }}$ Century Parisians, 472 cubic centimeters; in present-day Parisians, 593 cubic centimeters. Additionally, in Germans today, this difference happens to be more than 700 cubic centimeters. The inequalities of skull capacity-hence, of intelligence-that exist in mankind therefore tend to constantly increase.
4. Height exerts an influence upon the volume of the skull and the weight of the brain, but this influence is minimal. By assembling into groups all individuals of the same height and obtaining the average weight of the brain of each group, one will discover that the average brain weight between the tallest and shortest group of individuals rarely attains a difference of 100 grams, whereas the difference in brain weight often amounts to 300 grams among individuals of the same height.
5. One's sex imparts a substantial influence upon the weight of the brain. A woman possesses a brain that is considerably less heavy than that of a man and this inferiority subsists when both are of equal age, height and weight. Various studies of female brains show that in the most civilized races, such as contemporary Parisians, there is a notable proportion of the female population whose skulls by their volume come nearer to those of the gorillas than to the most developed male skulls. In a general way the brain of a civilized woman much more resembles that of a man belonging to an inferior race than that of a civilized man.
6. The difference which exists between the brain weight, and by consequence the skull volume, of a man and a woman progressively increases as a people's level of civilization rises; so,
from the point of view of the mass of the brain and therefore of intelligence, the female tends to become more and more differentiated from the male. The difference that exists, for example, between the average skull volume of contemporary Parisian males and females is nearly double that which exists between males and females of inferior races or of certain vanished races, like the inhabitants of ancient Egypt.
7. Female skulls of superior races, where the role of women is of little account, are remarkably smaller than female skulls belonging to a great number of inferior races. Whereas the average skull capacity of male Parisians ranks among the largest skulls known, the average skull capacity of female Parisians ranks among the smallest female skulls observed, barely exceeding the skull capacity of the women of New Caledonia.
8. With respect to subjects possessing the same skull circumference but who might present differences in skull volume upwards of 200 cubic centimeters, this is easily understood when one recalls that several factors, notably the height of the skull, may account for the variation of the volume for a given circumference. Hence, when one works on a series of skulls, one soon realizes that a 1 centimeter increase in the total circumference of the skull corresponds to an expansion of the volume that fluctuates over a 100 cubic centimeter range. The known properties of spherical bodies immediately suggest that as the circumference is increased by 1 centimeter upon a small head or upon a large head, the respective increase in the volume must be a little less or somewhat greater to that which I have indicated.
9. The comparative study of the graphs of the skull circumference with that of the head, as well as with the skull volume and brain weight, has made evident the relations existing between these different quantities and has rendered possible the construction of tables that, just by containing one of these known quantities, permits one to immediately determine the others whenever one operates upon a series. For example, one sees that among modern Parisians a head whose circumference is 57 centimeters corresponds to a skull whose circumference is 52
centimeters and volume is 1550 cubic centimeters. The weight of the brain contained in this skull will likely be 1350 grams.
10. There is invariably an unevenness in the development of the two halves of the brain, which is sometimes more developed on the right side, sometimes on the left, without one's level of intelligence or race seeming to have any apparent influence upon the location of this unevenness in development. However, this uneven development does not manifest itself in the same way in each of the parts of the skull.
11. The differences in skull capacity that one observes among the diverse categories of individuals of the same race do not appear to be attributable to causes other than the level of intelligence, in view of the fact that when these categories are sufficiently well-represented, they each obviously include just as many individuals of the same height and weight. With measurements effectuated upon 1,200 heads of living Parisians, I have offered proof that from the point of view of the volume of their heads they rank, from largest to smallest, in the following order: 1) scientists and men of letters; 2) middle-class Parisians; 3) nobles of ancient families; 4) domestics; 5) peasants.

[^0]:    * Note: All the skulls used for the construction of this table originate from the collections of the Museum of Anthropology, except for some of the Australian aborigine skulls, belonging to a foreign collection, that were measured during the Exposition by the Anthropology lab's very able assistant, Doctor Callamand.

